General Physics II: Midterm Exam (113–1)

100 minutes, full mark = 50

Use of your notebooks/memos/books:	Strictly forbidden.
Use of your mobile etc. & Internet:	Strictly forbidden.
Discussion with other attending students:	Strictly forbidden.

Administrative Remarks

- Write your name and student ID on the answer sheet. Put your student ID on the desk.
- Allowed on your desk: student ID card (required), pens/pencils, correction tools (eraser etc.), rulers, and drinks. **Other items must be stored in your bags.**
- You cannot wear watches nor electronic devices. **You cannot have them even in your pockets**.
- **After 9:10, the following actions are considered cheating. You may immediately lose your credit.**
  - If non-allowed items (pen cases, foods, poaches, etc.) are found on desks.
  - If you have textbooks, mobile phones, tablets, or PC, if they are not stored in your bags, or if you use them. They must be in your bags even after you submit your answer sheets.
- Breaks are not allowed in principle. After 10:00, you may leave after submission. In case of health problems or other issues, call the TA or lecturer.
- *Any form of academic dishonesty, including chats, additions/corrections after the period, and using your phones, will be treated by NSYSU "Academic Regulations."*

Scientific Remarks

- Show your calculations or thought process for **partial mark**!
- Use English, where mistakes are tolerated. Meanwhile, scientific mistakes are not tolerated.
  - Provide appropriate **units** properly. Handle **significant digits** properly.
  - Clearly distinguish **vectors** (by writing $\vec{E}$, $\vec{x}$ or $\mathbb{E}$, $\mathbb{x}$) from scalars (E, x).
- If you find any errors or issues in the questions, explain them on your answer sheet, make necessary adjustments on the question, and answer accordingly.
- You may use the following symbols and values without definition/declaration.

standard acceleration of gravity	g	$= 9.8\,\mathrm{m/s^2}$
Newtonian constant of gravitation	G	$= 6.7\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}$
elementary charge	e (or $\lvert e\rvert$)	$= 1.6\times10^{-19}\,\mathrm{C}$
permittivity of free space	ϵ_0	$= 8.9\times10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$
permeability of free space	$\mu_0 = \left(\epsilon_0 c^2\right)^{-1}$	$= \pi\times4.0\times10^{-7}\,\mathrm{N/A^2}$
Coulomb constant	$k_e = \left(4\pi\epsilon_0\right)^{-1}$	$= 9.0\times10^{9}\,\mathrm{N\cdot m^2/C^2}$
speed of light in vacuum	c	$= 3.0\times10^{8}\,\mathrm{m/s}$
Avogadro's number	N_A	$= 6.0\times10^{23}/\mathrm{mol}$
masses of protons and electrons	m_p, m_e	$= 1.7\times10^{-27}\,\mathrm{kg}, 9.1\times10^{-31}\,\mathrm{kg}$

Unit vectors in the direction of the axes $(\vec{e_x}, \vec{e_y}, \vec{e_z})$ or $(\hat{e_x}, \hat{e_y}, \hat{e_z})$ or $(\hat{\imath}, \hat{\jmath}, \hat{k})$

$\vec{E}(\vec{x})$	electric field at $\vec{x}$	$\vec{B}(\vec{x})$	magnetic field (magnetic flux density) at $\vec{x}$
$V(\vec{x})$	electrostatic potential at $\vec{x}$	$\vec{F}_{XY}$	force exerted by X on Y

$\sqrt{2}\approx1.414$ $\quad\sqrt{3}\approx1.732$ $\quad\sqrt{5}\approx2.236$ $\quad\sqrt{7}\approx2.646$ $\quad\pi\approx3.142$ $\quad\mathrm{e}\approx2.718$

Answer all the problems.

[Part A] Fundamental concepts (11 points)

Here are some formulae we discussed in the class. Answer the questions about the quantities therein.

$$\Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{A}, \qquad Q = C\Delta V, \qquad R = \frac{\rho l}{A} = \frac{l}{\sigma A}, \qquad \vec{J} = \sigma \vec{E}, \qquad I = n|q||\vec{v}_{\mathrm{d}}|A.$$

(1) For each of the following quantities, give its name.

a) C b) R c) ρ d) Φ

(2) For each of the following quantities, give its unit.

a) C b) $\vec{E}$ c) Q d) Φ

e) resistivity f) current density

(3) The notation $|\ |$ is used with different meanings for $|q|$ and $|\vec{v}_{\mathrm{d}}|$ above. Explain the difference.

(4) Explain what the drift velocity $\vec{v}_{\mathrm{d}}$ is by describing its meaning or definition.

[Part B] Coulomb's law and Electric field (9 points)

Consider two different points, P (a, b, c) and R (x, y, z), in a three-dimensional space. A **positive** point charge Q is located at Point P. We consider the electric field $\vec{E}$ at Point R.

(1) Describe the vector $\overrightarrow{\mathrm{PR}}$ by using $\overrightarrow{\mathrm{OP}}$ and $\overrightarrow{\mathrm{OR}}$.

(2) Describe $|\overrightarrow{\mathrm{PR}}|$ by using a, b, c, x, y, and z.

(3) Describe $\vec{E}$ at Point R by using $\overrightarrow{\mathrm{PR}}$, $|\overrightarrow{\mathrm{PR}}|$, $\overrightarrow{\mathrm{RP}}$, $|\overrightarrow{\mathrm{RP}}|$, Q, k_e, and ϵ_0. (You do not have to use all of them.)

(4) Describe $\vec{E}$ at Point R by using a, b, c, x, y, z, Q, and k_e or ϵ_0.

(5) Let V_0 be the electrostatic potential level at infinity. Describe the electrostatic potential V at Point R by using V_0, a, b, c, x, y, z, Q, and k_e or ϵ_0.
(It is sufficient just to write down the expression; no need to derive the equation.)

[Part C] Gauss's law (15 points)

A small solid sphere with a radius 1.00 mm, made by a conductor, has a charge 3.14 μC. Its center is at the origin O(0, 0, 0) and no other charges are nearby.

(1) Describe how the electric charge distributes over the sphere, quantitatively if possible.

Now, imagine a cube of edge 2.0 m whose center is at the origin. We discuss the electric flux through the imaginary cube, where you can use $\epsilon_0 \approx 8.9 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$ and $3.14/8.9 \approx 0.35$.

(2) Find the electric flux through each face of the cube.

(3) Find the electric flux through the whole surface of the cube.

Let us define the coordinate: we assume the edges are parallel to the x-, y-, or z-axis. As its center is at the origin O, the corners of the cube are at ($\pm$1.0 m, $\pm$1.0 m, $\pm$1.0 m).

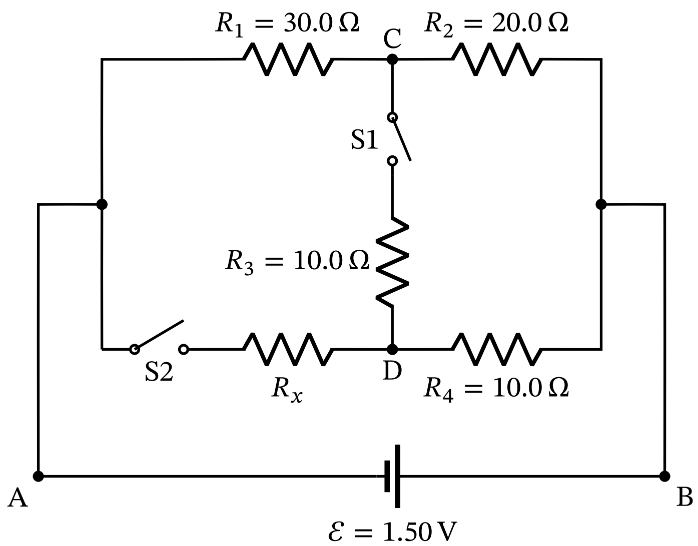
(4) Assume the sphere is not at O but at (1.0 cm, 2.0 cm, 0). Find which surface(s) of the cube has the largest electric flux and which surface(s) has the smallest electric flux, and explain why.

[The exam questions continue on the next page.]

[Part D] DC Circuit (15 points)

Consider the circuit below, made of two switches S1 and S2, a battery with $\mathcal{E} = 1.50\,\mathrm{V}$ and negligible internal resistance, and five resistors. Resistances of four resistors are known, but one of them has an unknown resistance R_x. We define the voltage at Point A, B, C, D as V_A, V_B, V_C, and V_D, respectively.

- At first, both switches are open ("off") as illustrated in the figure. Find **(1)** the current, **(2)** the voltage difference $V_C - V_A$, and **(3)** the power consumption of the battery.
- Then, the switch S1 is closed. Find **(4)** the equivalent resistance between A and B and **(5)** the voltage difference $V_D - V_A$. Also, **(6)** describe the current flow in the circuit.
- Finally, both switches are closed. Then, the current flowing through the resistor R_3 becomes precisely zero. **(7)** Find the resistance R_x. [Note: This is called Wheatstone bridge.]



The following are extra problems. Somewhat advanced but not very challenging.

[Part E] Extra Problem 1

Consider a straight wire made by a conductor. It has a linear charge density λ. Assuming it has an infinite length, discuss the electric field around the wire. [Hint: Use Gauss's law.]

[Part F] Extra Problem 2

Consider a parallel-plate capacitor with area $A = 0.0113\,\mathrm{m}^2$ and distance $d = 0.010\,\mathrm{m}$ between the plates and assume a voltage of $\Delta V = 3.0\,\mathrm{V}$ is applied. The charge stored in each capacitor plate is given by $\pm Q = \pm C\Delta V = \pm 0.030\,\mathrm{nC}$, where $C = \epsilon_0 A/d = 0.010\,\mathrm{nF}$.

(1) Calculate the energy stored in the capacitor.

(2) Show that those plates are attracting each other with a force $|\vec{F}| = \dfrac{Q^2}{2\epsilon_0 A} = 4.5 \times 10^{-9}\,\mathrm{N}$.

[This is the end of the exam questions.]