Term Exam 112–2 (Jun. 5, 2024)

100 minutes, full mark = 50

Use of your notebooks/memos/books:	You can bring-in only a sheet of A4 paper.
Use of your mobile etc. & Internet:	Strictly forbidden.
Discussion with other attending students:	Strictly forbidden.

Administrative Remarks

- Write your name and student ID on the answer sheet. Put your student ID on the desk.
- Allowed on your desk: student ID card (required), pens/pencils, correction tools (eraser etc.), rulers, drinks, and the brought-in A4 sheet.
- **Other items must be stored in your bags and the bags must be put away from you before 9:10am.** You cannot wear/have watches nor electronic devices, even in your pockets.
- **After 9:20, the following acts are regarded as cheating. You may immediately lose your credit.**
  - If non-allowed items (pen cases, foods, poaches, etc.) are found on desks.
  - If you have textbooks, mobile phones, tablets, or PC, if they are not stored in your bags, or if you use them. They must be in your bags even after you submit your answer sheets.
- Breaks are not allowed in principle. After 10:00, you may leave after submission. In case of health problems or other issues, call the TA or lecturer.
- *Any form of academic dishonesty, including chats, additions/corrections after the period, and using your phones, will be treated by NSYSU "Academic Regulations."*

Scientific Remarks

- Show your calculations or thought process for **partial mark**!
- Use English, where mistakes are tolerated. Meanwhile, scientific mistakes are not tolerated.
  - Provide appropriate **units** properly. Handle **significant digits** properly.
  - Clearly distinguish **vectors** (by writing $\vec{E}$, $\vec{x}$ or $\mathbb{E}$, $\mathbb{x}$) from scalars (E, x).
- If you find any errors or issues in the questions, explain them on your answer sheet, make necessary adjustments on the question, and answer accordingly.
- You may use the following symbols and values without definition/declaration.

standard acceleration of gravity	g	$= 9.8\,\mathrm{m/s^2}$
Newtonian constant of gravitation	G	$= 6.7\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}$
elementary charge	e (or $\lvert e\rvert$)	$= 1.6\times10^{-19}\,\mathrm{C}$
permittivity of free space	ϵ_0	$= 8.9\times10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$
permeability of free space	$\mu_0 = \left(\epsilon_0 c^2\right)^{-1}$	$= \pi\times4.0\times10^{-7}\,\mathrm{N/A^2}$
Coulomb constant	$k_e = \left(4\pi\epsilon_0\right)^{-1}$	$= 9.0\times10^{9}\,\mathrm{N\cdot m^2/C^2}$
speed of light in vacuum	c	$= 3.0\times10^{8}\,\mathrm{m/s}$
Avogadro's number	N_A	$= 6.0\times10^{23}/\mathrm{mol}$
masses of protons and electrons	m_p, m_e	$= 1.7\times10^{-27}\,\mathrm{kg}, 9.1\times10^{-31}\,\mathrm{kg}$

Unit vectors in the direction of the axes $(\vec{e_x}, \vec{e_y}, \vec{e_z})$ or $(\hat{e}_x, \hat{e}_y, \hat{e}_z)$ or $(\hat{\imath}, \hat{\jmath}, \hat{k})$

$\vec{E}(\vec{x})$ electric field at $\vec{x}$ $\vec{B}(\vec{x})$ magnetic field (magnetic flux density) at $\vec{x}$
$V(\vec{x})$ electrostatic potential at $\vec{x}$

$\sqrt{2}\approx1.414$ $\sqrt{3}\approx1.732$ $\sqrt{5}\approx2.236$ $\sqrt{7}\approx2.646$ $\pi\approx3.142$ $\mathrm{e}\approx2.718$

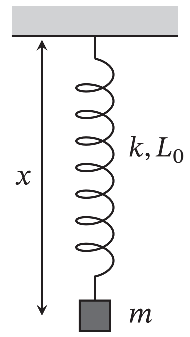
Answer all the problems.

[Part I] Review Problems (15 points)

(1) Consider a solid sphere made of a conductor. It is in xyz-space, its center is at $(0,0,0)$, and its radius is A. It carries a total charge $Q > 0$.

- **(A)** Discuss how the charge distributes over the sphere.
- **(B)** Calculate $|\vec{E}|$ inside and outside the sphere.

(2) A small object with mass m is attached to the bottom end of a hanging vertical spring, whose spring constant is k and equilibrium length is L_0. At $t = 0$, the object is at rest and the spring is at equilibrium. Therefore, the motion of this block is described by

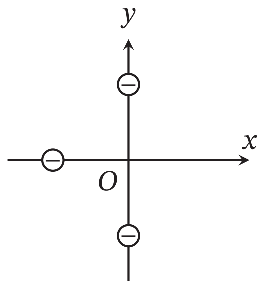


$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t) = -k(x(t) - L_0) + mg, \quad \frac{\mathrm{d}}{\mathrm{d}t}x(0) = 0, \quad x(0) = L_0,$$

where air resistance is neglected. Discuss the motion of this object.

[Hint: Define $f(t) := x(t) - (L_0 + mg/k)$ to simplify the differential equation; solve it to find the amplitude, period, and frequency of the motion. Discuss energy conservation.]

(3) Three **negative** charges, each with charge $-q$, are fixed in the xy-plane at positions $(0, \pm a)$ and $(-a, 0)$, where $q > 0$ and $a > 0$. The reference level of the electric potential is set to V_0 at infinity. Calculate electric field and electric potential at (s, t).



[Hint: If you are not confident in your answer, solve the problem for $(0,0)$ instead of (s,t) for a partial mark.]

[Part II] Maxwell's Equations (8 points)

Electromagnetism is described by Maxwell's equations. In integral form, they are given by

Ⓐ $\displaystyle\oint_S \vec{E}\cdot\vec{n}\,\mathrm{d}A = \frac{q}{\epsilon_0},$ Ⓑ $\displaystyle\oint_S \vec{B}\cdot\vec{n}\,\mathrm{d}A = 0,$

Ⓒ $\displaystyle\oint_C \vec{E}\cdot\mathrm{d}\vec{s} = -\frac{\mathrm{d}}{\mathrm{d}t}\Phi_B,$ Ⓓ $\displaystyle\oint_C \vec{B}\cdot\mathrm{d}\vec{s} = \mu_0 I + \mu_0\epsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\Phi_E.$

(1) Describe the meaning of $\vec{n}$ and q in Ⓐ, paying attention the meaning of S and direction of $\vec{n}$.

(2) Write down the units of $\vec{B}$, I, and Φ_E appearing in Ⓓ.

(3) Which equation describes Faraday's law?

[The exam questions continue on the next page.]

[Part III] Vector-or-Scalar-or-Not Reincarnated (4 points)

Select all scalar quantities from the following words. (Answer by the labels, not words.)

(1) critical temperature	**(5)** capacitor	**(9)** superconductivity	**(13)** solenoid
(2) linear charge density	**(6)** resistance	**(10)** resistivity	**(14)** Lenz
(3) electric potential	**(7)** magnetic field	**(11)** permittivity	**(15)** electron
(4) dielectric constant	**(8)** Lorentz force	**(12)** permeability	**(16)** dielectrics

[Part IV] Electric Current (10 points)

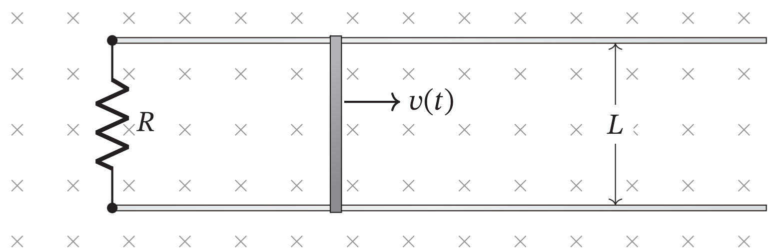
A copper wire with a cross section $2.5\,\mathrm{mm}^2$ carries a current of $4.2\,\mathrm{A}$. We calculate the drift velocity of the free electrons, assuming each copper atom contributes one free electron to the current.

(1) Explain "drift velocity" in simple terms so that high school students can understand.

(2) With the approximation that $1\,\mathrm{mm}^3$ of copper contains 8.4×10^{19} copper atoms, calculate the magnitude of the drift velocity, $|\vec{v}_\mathrm{d}|$, of the free electrons.

(3) Describe the direction and magnitude of the magnetic field at $1.0\,\mathrm{m}$ from the wire.

[Part V] Magnetic Force Acting on a Sliding Bar (13 points)



As in the figure, a conducting bar is moving on two parallel rails under a uniform magnetic field. The magnetic field is present in the whole space, directed into the sheet, and has a magnitude B. The rails are separated by the distance L, sufficiently long, and frictionless. The bar has mass m and negligible resistance; it is perpendicular to the rails, moves rightward, and does not leave the rails during this experiment. The left ends of the rails are connected to a resistor with resistance R, while the right ends, which are in the direction of the bar's motion, are left open and unconnected.

At $t = 0$, the bar has a speed $v(0) = v_0 > 0$. Answer the following questions.

(1) Calculate the kinetic energy $E_\mathrm{K}(t)$ of the bar at $t = 0$.

(2) Describe the direction and magnitude of the current induced in the loop, $I(t)$, for $t > 0$, using $v(t)$ and the values given in the problem.

(3) Calculate the power $P(t)$ dissipated in the resistor.

(4) Discuss the magnetic force $\vec{F}_B(t)$ acting on the bar.

(5) Find $v(t)$ by solving a differential equation. Find $E_\mathrm{K}(t)$ and $P(t)$. Discuss energy conservation.

[The exam questions continue on the next page.]

[Part VI] **Extra Problem** (unlimited points)

> This is a challenging problem, but you may simplify the situation for a partial mark.

A proton is moving under a uniform but time-dependent magnetic field. Discuss its motion.

Hints

- This is (in fact) extremely tough; you may face this problem in graduate school. To simplify,
  - Assume the speed is much smaller than the speed of light, so that you can ignore relativistic effect.
  - First, analyze the case with $|\vec{B}|$ being constant. (This is an intermediate-level problem for you.) Then, try to analyze more general case such as $f(t) = at + b$.
- Neglect forces other than Lorentz force. Assume there is no other particle in the neighborhood.
- You can assume the proton does not escape from the magnetic field.
- You can take the xyz-axes so that, with $\vec{r}(t)$ and $\vec{v}(t)$ being the velocity of the proton,

$$\vec{B}(x, y, z, t) = f(t)\vec{e_z} = \begin{pmatrix} 0 \\ 0 \\ f(t) \end{pmatrix}, \qquad \vec{v}(t = 0) = \begin{pmatrix} v_x \\ 0 \\ v_z \end{pmatrix}.$$

[This is the end of the exam questions.]