Midterm Exam 112–2 (Apr. 10, 2024)

100 minutes, full mark = 50

Use of your notebooks/memos/books:	You can bring-in only a sheet of A4 paper.
Use of your mobile etc. & Internet:	Strictly forbidden.
Discussion with other attending students:	Strictly forbidden.

Administrative Remarks

- Write your name and student ID on the answer sheet. Put your student ID on the desk.
- Allowed on your desk: student ID card (required), pens/pencils, correction tools (eraser etc.), rulers, drinks, and the brought-in A4 sheet. **Other items must be stored in your bags.**
- **After 9:20, the following acts are regarded as cheating. You may immediately lose your credit.**
  - If non-allowed items (pen cases, foods, poaches, etc.) are found on desks.
  - If your mobile phones, tablets, or PC are not stored in your bags, or if you use them. They must be in your bags even after you submit your answer sheets.
- Breaks are not allowed in principle. After 10:00, you may leave after submission. In case of health problems or other issues, call the TA or lecturer.
- *Any form of academic dishonesty, including chats, additions/corrections after the period, and using your phones, will be treated by NSYSU "Academic Regulations."*

Scientific Remarks

- Include your calculations or thinking process for **partial mark**!
- Use English, where mistakes are tolerated. Meanwhile, scientific mistakes are not tolerated.
  - Provide appropriate **units** if necessary.
  - Handle **significant digits** properly.
  - Clearly distinguish **vectors** (by writing $\vec{E}$, $\vec{x}$ or $\mathbb{E}$, $\mathbb{x}$) from scalars (E, x).
- If you notice any errors/issues in the questions, explain the error on your answer sheet, suitably adjust the question, and answer the corrected question.
- You may use the following symbols and values without definition/declaration.

standard acceleration of gravity	g	$= 9.8\,\mathrm{m/s^2}$
Newtonian constant of gravitation	G	$= 6.7\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}$
elementary charge	e (or $\lvert e\rvert$)	$= 1.6\times10^{-19}\,\mathrm{C}$
permittivity of free space	ϵ_0	$= 8.9\times10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$
permeability of free space	$\mu_0 = \left(\epsilon_0 c^2\right)^{-1}$	$= 1.3\times10^{-6}\,\mathrm{N/A^2}$
Coulomb constant	$k_e = \left(4\pi\epsilon_0\right)^{-1}$	$= 9.0\times10^{9}\,\mathrm{N\cdot m^2/C^2}$
speed of light in vacuum	c	$= 3.0\times10^{8}\,\mathrm{m/s}$
Avogadro's number	N_A	$= 6.0\times10^{23}/\mathrm{mol}$
masses of protons and electrons	m_p, m_e	$= 1.7\times10^{-27}\,\mathrm{kg}$, $9.1\times10^{-31}\,\mathrm{kg}$

Unit vectors in the direction of the axes $(\vec{e_x}, \vec{e_y}, \vec{e_z})$ or $(\hat{e}_x, \hat{e}_y, \hat{e}_z)$ or $(\hat{\imath}, \hat{\jmath}, \hat{k})$

$\vec{E}(\vec{x})$ electric field at $\vec{x}$ $\vec{B}(\vec{x})$ magnetic field (magnetic flux density) at $\vec{x}$
$V(\vec{x})$ electrostatic potential at $\vec{x}$

$\sqrt{2}\approx 1.414$ $\sqrt{3}\approx 1.732$ $\sqrt{5}\approx 2.236$ $\sqrt{7}\approx 2.646$ $\pi\approx 3.142$ $\mathrm{e}\approx 2.718$

Answer all the problems. You can use the formula

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, \quad \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}.$$

[Part I] Wave (12 points)

(1) Consider an electromagnetic wave with speed 3×10^8 m/s and frequency 8×10^{14} Hz. Discuss its properties (period, angular wave number, etc.) and anything related to it. In this problem, you can approximate $\pi \approx 3$.

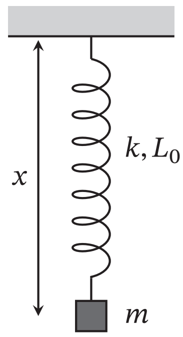
(2) Derive the formula for beating frequency, $f_{\mathrm{beat}} = |f_1 - f_2|$.

[Part II] Oscillation (8 points)

As in the right figure, a small object with mass m is attached to the bottom end of a hanging vertical spring, whose spring constant is k and equilibrium length is L_0. At $t = 0$, the object is at rest and the spring is at equilibrium. Therefore, the motion of this block is described by, with the air-resistance force $F_{\mathrm{air}} \approx bv(t)$,

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t) = -k(x(t) - L_0) + mg - F_{\mathrm{air}}, \quad \frac{\mathrm{d}}{\mathrm{d}t}x(0) = 0, \quad x(0) = L_0,$$

Discuss the motion of this object.



[Hint: Neglect F_{air} by setting $b = 0$ and define $f(t) \coloneqq x(t) - (L_0 + mg/k)$ to simplify the differential equation; solve it to find the amplitude, period, and frequency of the motion under no air resistance. Discuss energy conservation. You can further try to provide a qualitative discussion under air resistance.]

[Part III] Conceptual Questions I (7 points)

(1) Write down the unit for each of the following quantities.

- **(A)** intensity of a periodic sound wave
- **(B)** angular wave number
- **(C)** linear charge density
- **(D)** electric flux

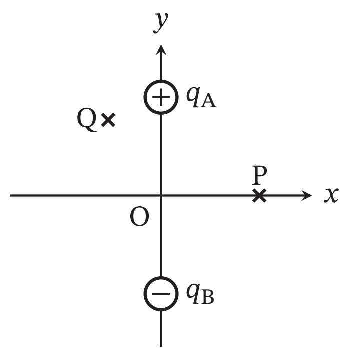
(2) Fill the six blanks in the sentences below.

- The highest point of a wave is called ___(E)___ and the lowest point is called ___(F)___.
- In standing waves, a point of zero amplitude is called ___(G)___, while a point of maximum displacement is called ___(H)___.
- ___(I)___'s law states that the net flux through any ___(J)___ surface surrounding a point charge q is given by q/ϵ_0.

[The exam questions continue on the next page.]

[Part IV] Conceptual Questions II (8 points)

As in the figure, two point charges are located on xy-plane; they have $q_A = +1\,\text{nC}$ and $q_B = -1\,\text{nC}$, respectively. The reference level of electrostatic potential V is set zero at infinity.



(1) Provide a sketch of the electric field lines.

(2) Fill the four blanks in the sentences below.

- This set-up is called an ____(A)____. [Hint: two words]
- V at P is ____(B)____ and V at Q is ____(C)____. [Choice: positive/negative/zero]
- $|\vec{E}|$ at P is ____(D)____ than $|\vec{E}|$ at Q. [Choice: larger/smaller]

(3) Fill the four blanks in the sentences below. You can choose from the eight cardinal directions such as "east", "northwest", and "southeast", assuming the positive y-direction points north.

- Because $|q_A| = |q_B|$, the direction of $\vec{E}$ at P is ____(E)____.
- If $q_A = 4\,\text{nC}$ and $q_B = -1\,\text{nC}$, the direction of $\vec{E}$ at P becomes ____(F)____. Under this set-up, consider $\vec{E}$ at very far points ($R \gg |\overrightarrow{\text{OP}}|$), where its magnitude $|\vec{E}|$ is tiny but non-zero. The direction of $\vec{E}$ at $(-R, 0)$ is ____(G)____, while the direction of $\vec{E}$ at $(0, -R)$ is ____(H)____.

[Part V] Electric Field (15 points)

Consider xyz-space. A point charge $+Q$ exists at the origin $O = (0, 0, 0)$, where $Q > 0$.

(1) Find electric field $\vec{E}(x, y, z)$.

(2) Let V_0 be the electrostatic potential level at infinity. Find electrostatic potential $V(x, y, z)$.

Now, instead of a point charge, consider a solid sphere of radius A carrying a total charge $Q > 0$. We assume it is made of an insulator, it has a uniform volume charge density $\rho = 3Q/4\pi A^3$, and its center is at $(0, 0, 0)$.

(3) Find the magnitude of the electric field, $|\vec{E}(x, y, z)|$, outside the sphere ($\sqrt{x^2 + y^2 + z^2} > A$).

(4) Find the magnitude of the electric field, $|\vec{E}(x, y, z)|$, inside the sphere ($\sqrt{x^2 + y^2 + z^2} < A$).

(5) What if the solid sphere is made of a conductor? Discuss the charge distribution and $|\vec{E}|$ inside and outside the sphere, assuming the same net charge $Q > 0$ and radius A.

[Part VI] Extra Problem (unlimited points)

This is a challenging problem for motivated students, but qualitative discussion is not difficult.

Three positive charges are on x-axis. Two of them are fixed. The other one can move freely without friction on x-axis but not in other directions. At $t = 0$, the unfixed charge is at rest and located at the midpoint of the other two fixed charges. Describe its motion after $t = 0$. [Hint: Draw $V(x)$.]

[This is the end of the exam questions.]