(calculation sheet)

Term Exam 111–2 (June 1, 2023)

100 minutes, full mark = 50

Use of your notebooks/memos/books:	You can bring-in only a sheet of A4 paper.
Use of your mobile etc. & Internet:	Not allowed.
Discussion with other attending students:	Not allowed.

Administrative Remarks

- Write your correct name and student ID on the answer sheet. Put your student ID on the desk.
- Allowed on your desk: student ID card (required), pens/pencils, correction tools (eraser etc.), rulers, drinks, the brought-in A4 sheet, and watches.
- Other things such as pen cases, foods, and poaches must be stored in your bags. In particular, **phones/tablets/books must be stored in your bags while you are in the room (even after submission). If you use them in the room, you may immediately lose your credit.**
- Breaks are not allowed in principle, but you may leave earlier after submission. In case of health problems or other issues, ask TA or lecturer.
- *Any form of academic dishonesty, including chats, additions/corrections after the period, and using your phones, will be treated by NSYSU "Academic Regulations."*

Scientific Remarks

- No penalty for wrong answers *in this exam*, so try to write something. Include your calculations or thinking process for **partial mark**!
- Use English, where mistakes are torelated. Meanwhile, scientific mistakes are not tolerated.
  - Provide appropriate units, if necessary.
  - Clearly distinguish vectors (by writing $\vec{E}$, $\vec{x}$ or $\mathbb{E}$, $\mathbb{x}$) from scalars (E, x).
- If you notice any errors/issues in the questions, explain the error on your answer sheet, suitably adjust the question, and answer the corrected question.
- You may use the following symbols and values without definition/declaration.

elementary charge	e (or $\lvert e\rvert$)	$= 1.6\times10^{-19}\,\mathrm{C}$
permittivity of free space	ϵ_0	$= 8.9\times10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$
permeability of free space	$\mu_0 = \dfrac{1}{\epsilon_0 c^2}$	$= 1.3\times10^{-6}\,\mathrm{N\,A^{-2}}$
Coulomb constant	$k_e = \dfrac{1}{4\pi\epsilon_0}$	$= 9.0\times10^{9}\,\mathrm{N\cdot m^2/C^2}$
speed of light in vacuum	c	$= 3.0\times10^{8}\,\mathrm{m/s}$
Avogadro's number	N_A	$= 6.0\times10^{23}\,/\mathrm{mol}$
masses of protons and electrons	m_p, m_e	$= 1.7\times10^{-27}\,\mathrm{kg}$, $9.1\times10^{-31}\,\mathrm{kg}$

Unit vectors in the direction of the axes $(\vec{e_x}, \vec{e_y}, \vec{e_z})$ or $(\hat{e_x}, \hat{e_y}, \hat{e_z})$ or $(\hat{\imath}, \hat{\jmath}, \hat{k})$

$\vec{E}(\vec{x})$	electric field at $\vec{x}$
$\vec{B}(\vec{x})$	magnetic field (magnetic flux density) at $\vec{x}$
$V(\vec{x})$	electrostatic potential at $\vec{x}$

[A] Review Questions (10 points)

Consider three different points A($\vec{a}$), B($\vec{b}$), and X($\vec{x}$), where $\vec{a} = \overrightarrow{\mathrm{OA}}$ and so on (position vectors). First, let us think a situation that a positive point charge $+q$ exists at point A.

(1) Describe $\vec{E}(\vec{x})$ [electric field at X] by using $\vec{a}$, $\vec{x}$, q, and $4\pi\epsilon_0$.

(2) Let V_0 be the electrostatic potential level at infinity. Describe electrostatic potential $V(\vec{x})$.

Let us think another situation, where a positve point charge $+q$ exists at point A and a negative point charge $-q$ exists at point B.

(3) Describe $\vec{E}(\vec{x})$ and $V(\vec{x})$ by using $\vec{a}$, $\vec{b}$, $\vec{x}$, q, and $4\pi\epsilon_0$.

(4) Let P($\vec{p}$) be the middle point of A and B, which means $\overrightarrow{\mathrm{AP}} = \overrightarrow{\mathrm{PB}}$. Calculate $\vec{E}$ and V at P. [Hint: Express $\vec{p}$ by $\vec{a}$ and $\vec{b}$. Do you recall how to express $\overrightarrow{\mathrm{AP}}$ by $\vec{a}$ and $\vec{p}$?]

Electric current $\vec{I}$ is usually described by $\vec{I} = nqA\vec{v}_\mathrm{d}$, where q is the charge of each current carrier, A is the cross-sectional area of the current, and $\vec{v}_\mathrm{d}$ is the drift velocity of the current carriers.

(5) What is n? Explain in 5–10 words.

(6) Describe the meaning of the drift velocity in one or a few sentences.

[B] Maxwell's Equations (10 points)

Electromagnetism is described by Maxwell's equations. In integral form, they are given by

$$\text{(A)}\quad \oint_S \vec{E}\cdot \mathrm{d}\vec{A} = \frac{q}{\epsilon_0}, \qquad \text{(C)}\quad \oint_S \vec{B}\cdot \mathrm{d}\vec{A} = 0,$$

$$\text{(B)}\quad \oint_C \vec{E}\cdot \mathrm{d}\vec{s} = -\frac{\mathrm{d}}{\mathrm{d}t}\Phi_B, \qquad \text{(D)}\quad \oint_C \vec{B}\cdot \mathrm{d}\vec{s} = \mu_0 I + \mu_0\epsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\Phi_E.$$

(1) Explain the meaning of q in (A) in a few sentences, paying attention to the meaning of S.

(2) Explain the meaning of I in (D) in a few sentences, paying attention to the meaning of C.

(3) The following laws/facts are closely related to one of the four equations (A)–(D). For each statement, choose one equation that is **most closely** related to it.

- **(a)** Gauss's law of magnetism.
- **(b)** Ampère's law, i.e., the formula to get $\vec{B}(\vec{x})$ from a straight infinite current.
- **(c)** Faraday's law, i.e., the law for electromagnetic induction.
- **(d)** (We can think that) currents generate magnetic fields.
- **(e)** There are electric charges in this universe.
- **(f)** There are no "magnetic charges" (magnetic monopoles) in this universe.
- **(g)** We can define electric potential if the system is static.

(4) The following laws/concepts are closely related to **two** of (A)–(D). For each statement, choose **two equations** that are **most closely** related to it.

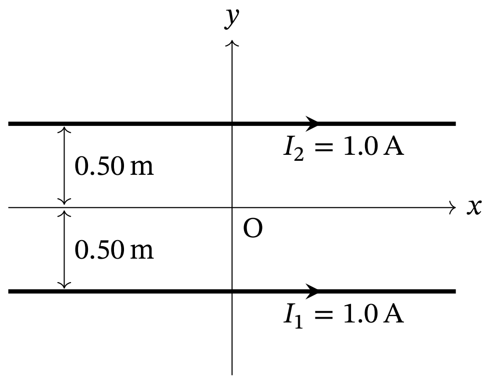
- **(a)** Biot–Savart law, i.e., the formula to get $\vec{B}(\vec{x})$ from current.
- **(b)** Light.
- **(c)** Inductors (coils).

[The exam questions continue on the next page.]

(calculation sheet)

(calculation sheet)

[C] Force between Currents (10 points)



As in the right figure, two long parallel wires are located on xy-plane. They are separated with a distance $d = 1.0\,\mathrm{m}$ and parallel to x-axis, which is at the middle of the wires. They carry steady currents in the same direction, rightward in the figure. Namely, the current I_1 passes at the point $(0, -0.50\,\mathrm{m}, 0)$, I_2 passes the point $(0, +0.50\,\mathrm{m}, 0)$, and both are in positive-x direction.

Now, we set $I_1 = I_2 = 1.0\,\mathrm{A}$. Answer the following questions about this system; we can assume the wires are infinite and the currents are static.

(1) We usually use the right-handed coordinate system for xyz-space. What is the direction of z-axis in the figure? (Answer "the direction into the sheet" or "the direction out of the sheet.")

(2) Magnetic field $\vec{B}$ is given by the sum of the magnetic field generated by I_1, which we call $\vec{B}_1$, and the magnetic field generated by I_2, which we call $\vec{B}_2$. Namely, $\vec{B}(\vec{x}) = \vec{B}_1(\vec{x}) + \vec{B}_2(\vec{x})$. Calculate $\vec{B}_1$, $\vec{B}_2$, and $\vec{B}$ at the origin O(0,0,0). [Hint: Do not forget to specify directions.]

We calculate the force between the wires. As the wires are infinite, the total force is infinite. So we focus on a segment $\Delta l = 1\,\mathrm{m}$ of the upper wire (with I_2) and calculate the force on the segment.

(3) Calculate $\vec{B}_1$ at the position of the segment Δl.

(4) The force acting on the segment is given by $\vec{F} = (\vec{I}_2 \times \vec{B}_1)\Delta l$, where $\vec{B}_1$ is the answer of the previous question, the direction of $\vec{I}_2$ is taken to be the direction of the current (i.e., positive-x), and $|\vec{I}_2| = I_2$. Calculate $\vec{F}$.

(5) Explain how to derive the formula $\vec{F} = (\vec{I}_2 \times \vec{B}_1)\Delta l$ from the definition of current ($\vec{I} = nqA\vec{v}_\mathrm{d}$) and the formula of the Lorentz force. [Hint: First, write down the Lorentz-force formula for partial mark.]

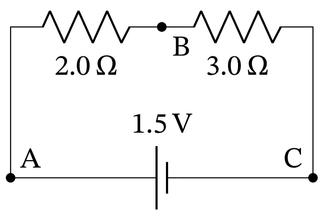
[D] DC Circuits (10 points)

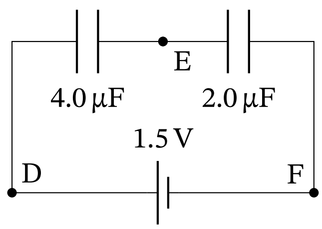
Consider the two circuits given below; resistors and capacitors are connected to an 1.5 V battery for a while, whose resistance and capacitance are given in the figure. We neglect the resistance of the circuit wire and the internal resistance of the battery. We denote the voltage at point A by V_A, etc.

(1) Calculate the voltage differences $V_\mathrm{A} - V_\mathrm{B}$, $V_\mathrm{B} - V_\mathrm{C}$, $V_\mathrm{D} - V_\mathrm{E}$, and $V_\mathrm{E} - V_\mathrm{F}$.

(2) Calculate the power consumption of each resistor.

(3) Calculate the charge on each capacitor and the energy stored in each capacitor.





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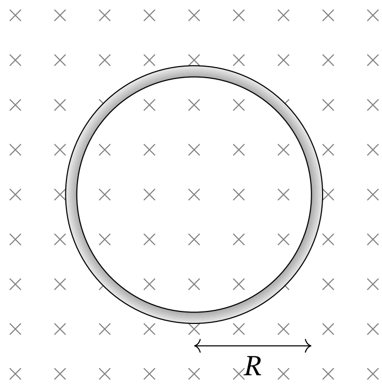
[E] Electromagnetic Induction (10 points)



As in the right figure, a circular loop of wire of radius R is placed in a magnetic field $\vec{B}$. The magnetic field is uniform, directed toward the sheet, and perpendicular to the plane of the loop. The magnetic field has a magnitude B_0 at $t = 0$ and decreases at a constant rate K in the time interval between $t = 0$ and t_1. Namely, for $0 \le t \le t_1$,

$$|\vec{B}(t)| = B_0 - Kt.$$

The direction of $\vec{B}$ is unchanged because $K < B_0/t_1$.

Because of the change of magnetic field, an emf was induced. We will step-by-step analyze the magnitude and direction of the induced emf based on one of Maxwell's equations,

$$\text{(B)} \quad \oint_C \vec{E} \cdot \mathrm{d}\vec{s} = -\frac{\mathrm{d}}{\mathrm{d}t}\Phi_B.$$

First, let us consider the right-hand side of the equation (B). The magnetic flux is given by

$$\Phi_B = \int_S \vec{B} \cdot \vec{n}\,\mathrm{d}A.$$

(1) We take S to be the area enclosed by the loop, which is a disk with radius R. We also take the direction of $\vec{n}$ to be the same direction as $\vec{B}$ (toward the sheet). Calculate Φ_B at $t = 0$.

(2) Calculate the right-hand side of (B).

Next, we consider the left-hand side. We can assume that the induced electric field $\vec{E}$ is along the wire loop and has the same magnitude on any point of the wire, which means we can set

$$\vec{E}(\text{at the wire}) = E_\theta\, \vec{e_\theta},$$

where $\vec{e_\theta}$ is the unit vector defined at each point on the loop so that it directs counterclockwise along the loop. In particular, $E_\theta > 0$ if $\vec{E}$ is counterclockwise, and $E_\theta < 0$ if $\vec{E}$ is clockwise.

(3) Explain why the direction of $\vec{E}$ is along the wire loop and why $|\vec{E}|$ is the same on any point of the wire. [Hint: Gauss's law and symmetry.]

(4) Describe how we should choose C in this case.

(5) Calculate the left-hand side of (B); you will use E_θ and R to express the result.

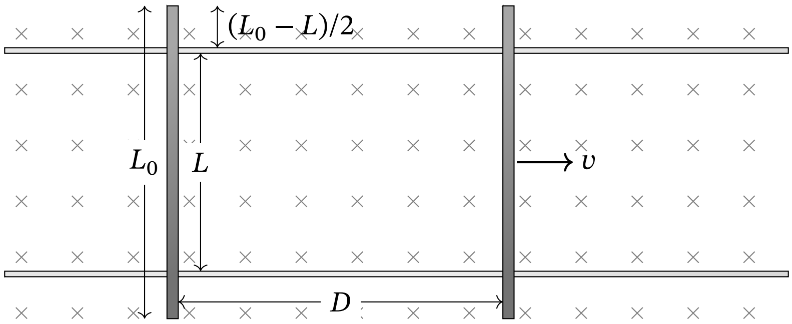
Now, let us suppose $R = 12.0\,\text{cm}$ and $K = 0.0500\,\text{T/s}$.

(6) Describe the direction and magnitude of the induced emf.

[The exam questions continue on the next page.]

[F] Extra Problem (unlimited points)

This is a challenging problem if you want to discuss quantitatively. However, qualitative (= non-quantitative) discussion of this system is at the basic level, so you should try. If you write something, you may earn something. Wrong answers receive no penalty in this exam!



As in the above figure, two rods made by conductor are located on two very long parallel wires. The rods are cylindrical shape with radius d and length L_0 and made of a uniform metal with resistivity ρ and mass density ρ_m. Meanwhile, the wires have negligible mass and resistance.

The wires are fixed and separated with the distance L to each other, where $L < L_0$. The rods are put on the wires as in the figure, in the middle of the wires. The rods can move on the wires, where we can assume they are always perpendicular to the wires. A uniform magnetic field $\vec{B}$ is applied perpendicular to the system, directed into the sheet.

The system is set horizontally to the ground, so you do not have to consider gravity. You may ignore the friction between rods and wires, but you may take it into account.

An external agent (a nine-year-old boy) came to the apparatus and did the following activity:

1. First, he fixed the left rod to the wires (with rubber bands) to prevent it from moving. At this moment, the distance between the rods were D.
2. Then, from $t = t_1$ to t_2, he pulled the right rod to the right with a constant speed v as in the figure.
3. Then, at $t = t_2$, he stopped his work; he released the right rod with the speed v.
4. Finally, at $t = t_3$, he released the fixation (rubber bands) of the left rod to allow it to move. (The right rod was still moving at this moment.)

Discuss physics during this period.

[Hint: You can discuss whatever you want, such as mass of the rods or distance between the rods at $t = t_2$. Non-quantitative answers will receive some partial marks, so you should try to guess the direction of motional emf, what happened to the left rod at $t = t_3$, and so on. Notice that you are asked to deliver your thought to readers, as if you are writing a book.]

[Comment: Qualitative discussion of this system is a basic-level exercise, so Sho hopes you get some extra marks from this problem. Meanwhile, you will earn very high score if you successfully discuss quantitatively, such as magnitude of motional emf, the force exerted by him during $t_1 < t < t_2$, and how much work he did.]

[This is the end of the exam questions.]