Midterm Exam 111–2 (Apr. 6, 2023)

100 minutes, full mark = 50

Use of your notebooks/memos/books:	Not allowed.
Use of your mobile etc. & Internet:	Not allowed.
Discussion with other attending students:	Not allowed.

Administrative Remarks

- **Write your correct name and student ID** on the answer sheet.
- Allowed on your desk: pens/pencils, correction-tools (eraser etc.), rulers, drinks, and watches.
- Other things such as phones, pen cases, foods, books, and poaches must be stored in your bags.
- Breaks are not allowed in principle, but you may leave earlier after submission. In case of health problems or other issues, ask TA or lecturer.
- **Any form of academic dishonesty, including chats, additions/corrections after the period, and using your phones, will be treated** by NSYSU "Academic Regulations."

Scientific Remarks

- Include your calculations and thinking process in your answer **for partial credit!**
- Use English, where gramatical/linguistic mistakes are tolerated (forgiven/allowed).
- As in in-class quizzes, **scientific mistakes are not tolerated.** In particular,
  - Provide appropriate units, if necessary.
  - Clearly distinguish vectors (by writing $\vec{E}$, $\vec{x}$ or $\mathbb{E}$, $\mathbb{x}$) from scalars (E, x).
- You may use the following symbols and values without definition/declaration.

elementary charge	e (or $\lvert e\rvert$)	$= 1.6\times10^{-19}\,\mathrm{C}$
permittivity of free space	ϵ_0	$= 8.9\times10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$
permeability of free space	$\mu_0 = \dfrac{1}{\epsilon_0 c^2}$	$= 1.3\times10^{-6}\,\mathrm{N\,A^{-2}}$
Coulomb constant	$k_e = \dfrac{1}{4\pi\epsilon_0}$	$= 9.0\times10^{9}\,\mathrm{N\cdot m^2/C^2}$
speed of light in vacuum	c	$= 3.0\times10^{8}\,\mathrm{m/s}$
Avogadro's number	N_A	$= 6.0\times10^{23}\,\mathrm{/mol}$
masses of protons and electrons	m_p, m_e	$= 1.7\times10^{-27}\,\mathrm{kg}, 9.1\times10^{-31}\,\mathrm{kg}$

Unit vectors in the direction of the axes $(\vec{e_x}, \vec{e_y}, \vec{e_z})$ or $(\hat{e_x}, \hat{e_y}, \hat{e_z})$ or $(\hat{\imath}, \hat{\jmath}, \hat{k})$

$\vec{E}(\vec{x})$	electric field at $\vec{x}$
$\vec{B}(\vec{x})$	magnetic field (magnetic flux density) at $\vec{x}$
$V(\vec{x})$	electrostatic potential at $\vec{x}$

- If you notice any errors/issues in the problems, explain the error in your answer sheet, suitably adjust the problem, and answer the corrected problem. (You may also ask the lecturer but not recommended.)

[A] Coulomb's law and Electric field I (10 points)

A point charge q exists at point A. Consider a different point X. We define $\vec{a} = \overrightarrow{\mathrm{OA}}$ and $\vec{x} = \overrightarrow{\mathrm{OX}}$.

(1) Describe the vector $\overrightarrow{\mathrm{AX}}$ by using $\vec{a}$ and $\vec{x}$.

(2) Describe the unit vector with the direction of $\overrightarrow{\mathrm{AX}}$ by using $\vec{a}$ and $\vec{x}$.

(3) Describe electric field at X by using $\vec{a}$, $\vec{x}$, q, and $4\pi\epsilon_0$.

(4) Let V_0 be the electrostatic potential level at infinity. Describe electrostatic potential at X by using $\vec{a}$, $\vec{x}$, q, $4\pi\epsilon_0$, and V_0.

[B] Coulomb's law and Electric field II (16 points)

As in Fig. 1, two point charges with opposite sign are located:
$+q$ at point A$(d, 0, 0)$ and $-q$ at B$(-d, 0, 0)$, where $q > 0$ and $d > 0$.
We consider points P$(2d, 0, 0)$ and Q$(0, r, 0)$, where $r > 0$, as well as the origin O.

(1) What is the direction of electric field at O, P, and Q? Describe with English words and/or mathematical expressions for each point.

(2) Describe the Coulomb force $\vec{F}_{\mathrm{AB}}$ caused by the charge at A acting on the charge at B.

(3) Describe the vector $\overrightarrow{\mathrm{AQ}}$ by using (if necessary) $q, d, r, \vec{e_x}, \vec{e_y}$, and $\vec{e_z}$.

(4) Calculate electric field at point Q.

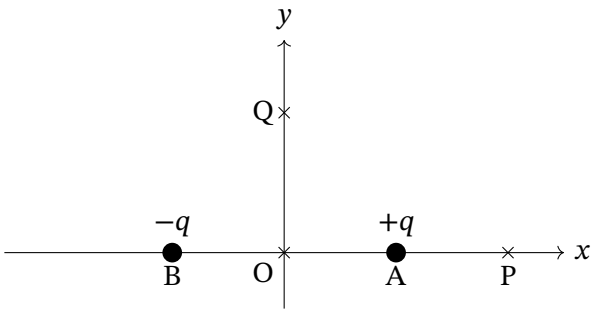

Fig. 1

[C] Gauss's law and Capacitor (17 points)

The capacitance C of a parallel-plate capacitor is (as a good approximation) given by $C = \epsilon_0 A/d$, where A is the area of the plates and d is the distance between the plates. We here derive this formula.

(1) What is the definition of the capacitance?

(2) We often express Gauss's law by $\displaystyle\oint_C \vec{E} \cdot \vec{n}\, \mathrm{d}A = \frac{Q}{\epsilon_0}$. Describe Gauss's law based on this expression. For example, you will first explain each symbol, explain the meaning of the integral, and discuss its physical implication.

(3) Imagine an infinite plate made by conductor. Assume it is uniformly charged, and the surface charge density is σ. Describe electric field caused by the plate.

(4) Derive the formula $C = \epsilon_0 A/d$.

[The exam questions continue on the next page.]

[D] Current and resistance (7 points)

As shown in Fig. 2, a resistor with $R = 10\,\Omega$ is connected to an 1.5 V battery with wire made by copper. We neglect the resistance of the copper wire and the internal resistance of the battery.

(1) These are statements on this circuit. For each statement, answer "T" if it is true (always correct) and "F" if it is false (incorrect or not precise).

- **(a)** The current runs clockwise (A→B→C).
- **(b)** Between A and B, the current is carried by free electrons of copper, where the free electrons are moving in the same direction.
- **(c)** The voltage at B is higher than the voltage at C.
- **(d)** The voltage at A is zero.

(2) Calculate the current.

(3) The copper wire between A and B is with a free charge density of 8.5×10^{19} electrons/mm^3, has a length 10 cm, and has a circular cross section with a radius of 1.0 mm. Discuss the drift velocity of the electrons.

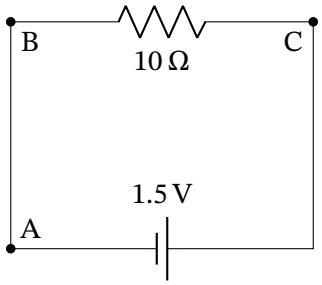

Fig. 2

[E] Extra Problem

This is a challenging problem for motivated students. Before trying this problem, please re-check your answers in the previous questions. Scientific mistakes will not be tolerated!

A cylinder with radius R and length L, made of insulator, is fully wrapped by thin aluminium foil with thickness d. Assume that the insulator cylinder has a uniform charge density ρ (> 0) (i.e., its total charge is $\pi R^2 L\rho$) and that the net charge of the aluminium foil is zero.

Under the approximation $L \gg R \gg d$, discuss electric field and charge distribution of this system together with your thinking process.

[Hint: The approximation $L \gg R$ means you can assume the cylinder is infinite and you do not have to consider its two ends. Find the symmetry of the system and its consequence.]

[This is the end of the exam questions.]