Midterm Exam 112–1 (Oct. 25, 2023)

100 minutes, full mark = 50

Use of your notebooks/memos/books:	Strictly forbidden.
Use of your mobile etc. & Internet:	Strictly forbidden.
Discussion with other attending students:	Strictly forbidden.

Administrative Remarks

- Write your name and student ID on the answer sheet. Put your student ID on the desk.
- Allowed on your desk: student ID card (required), pens/pencils, correction tools (eraser etc.), rulers, drinks, and watches. **Other items must be stored in your bags.**
- **After 9:20, the following acts are regarded as cheating. You may immediately lose your credit.**
  - If non-allowed items (pen cases, foods, poaches, etc.) are found on desks.
  - If your textbooks, notebooks, mobile phones, tablets, or PC are not stored in your bags, or if you use them. They must be in your bags even after you submit your answer sheets.
- Breaks are not allowed in principle. After 10:10, you may leave after submission. In case of health problems or other issues, call the TA or lecturer.
- *Any form of academic dishonesty, including chats, additions/corrections after the period, and using your phones, will be treated by NSYSU "Academic Regulations."*

Scientific Remarks

- Include your calculations or thinking process for **partial mark**!
- Use English, where mistakes are torelated. Meanwhile, scientific mistakes are not tolerated.
  - Provide appropriate **units**, if necessary.
  - Clearly distinguish **vectors** (by writing $\vec{E}$, $\vec{x}$ or $\mathbb{E}$, $\mathbb{x}$) from scalars (E, x).
- Respect **significant digits** if you can (no penalty). Bonus points are given for perfect treatment.
- If you notice any errors/issues in the questions, explain the error on your answer sheet, suitably adjust the question, and answer the corrected question.
- You may use the following symbols and values without definition/declaration.

standard acceleration of gravity	g	$= 9.8\,\mathrm{m/s^2}$
Newtonian constant of gravitation	G	$= 6.7\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}$
elementary charge	e (or $\lvert e\rvert$)	$= 1.6\times10^{-19}\,\mathrm{C}$
permittivity of free space	ϵ_0	$= 8.9\times10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$
permeability of free space	$\mu_0 = \left(\epsilon_0 c^2\right)^{-1}$	$= 1.3\times10^{-6}\,\mathrm{N/A^2}$
Coulomb constant	$k_e = \left(4\pi\epsilon_0\right)^{-1}$	$= 9.0\times10^{9}\,\mathrm{N\cdot m^2/C^2}$
speed of light in vacuum	c	$= 3.0\times10^{8}\,\mathrm{m/s}$
Avogadro's number	N_A	$= 6.0\times10^{23}\,\mathrm{/mol}$
masses of protons and electrons	m_p, m_e	$= 1.7\times10^{-27}\,\mathrm{kg}, 9.1\times10^{-31}\,\mathrm{kg}$

Unit vectors in the direction of the axes $(\vec{e_x}, \vec{e_y}, \vec{e_z})$ or $(\hat{e}_x, \hat{e}_y, \hat{e}_z)$ or $(\hat{\imath}, \hat{\jmath}, \hat{k})$

$\vec{E}(\vec{x})$ electric field at $\vec{x}$ $\vec{B}(\vec{x})$ magnetic field (magnetic flux density) at $\vec{x}$

$V(\vec{x})$ electrostatic potential at $\vec{x}$

$\sqrt{2}\approx 1.414$ $\sqrt{3}\approx 1.732$ $\sqrt{5}\approx 2.236$ $\sqrt{7}\approx 2.646$ $\pi\approx 3.142$ $\mathrm{e}\approx 2.718$

Answer **[Part A]**, **[Part B]**, and **[Part C]**. In addition, answer either **[Part D]** or **[Part E]**.

[Part A] Uniform Circular Motion (10 points)

Read the following text and answer the questions.

The Earth orbits around the Sun. Let's approximate it as uniform circular motion around the Sun, where the radius is $R = 1.5\times10^8$ km and the **(A)** is $T = 1$ year. We can find the speed v and the acceleration a. In particular, $a = R\omega^2$, where ω is called the **(B)** .

Thanks to Newton's second law, we can find the net force $\vec{F}$ exerted on the Earth. Its magnitude is given by $|\vec{F}| = mR\omega^2 = 3.5\times10^{22}$ N, where m is the mass of the **(C)** . The main origin of $\vec{F}$ is the gravitational force exerted by the Sun.

The unit "N" (newton) is one of the SI derived units, which are composed by the SI base units. Currently, the SI has seven base units, such as "kg", "mol", "cd", and "A".

(1) Fill the blanks **(A)**, **(B)**, and **(C)**. [Hint: **(B)** should be two words, and the others be a word.]

(2) Four out of seven SI base units are given in the text. What are the other three?

(3) Express "N" (newton) by the SI base units.

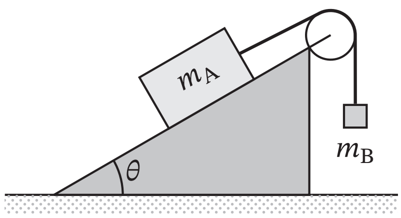
(4) Explain the direction of $\vec{F}$.

(5) Calculate v and ω. You may use 1 year $\approx 1.0\times\pi\times10^7$ s. (Physicists often use this approximation.)

[Part B] Forces and Motion (14 points)

As in the right figure, a wedge with an angle θ is fixed on the ground, Block A is on its slope, and Block B is connected to Block A by a wire through a pulley.

Block A has a mass m_A and can move only on the slope. Block B has a mass m_B and can move only vertically. Between Block A and the slope, the coefficient of kinetic friction is μ_k and the static-friction coefficient is μ_s. The wire has a negligible mass and does not stretch. Assume its tension is positive throughout this problem. Friction on the pulley is negligible.



When you see, Block A is moving up the slope with a speed v.

(1) List all the forces exerted on Block A. For each force, describe its direction. (You do not have to find their magnitude.)

(2) List all the forces exerted by Block A. For each force, describe its direction. (You do not have to find their magnitude.)

(3) Calculate the acceleration of Block A.

(4) Calculate the tension of the wire.

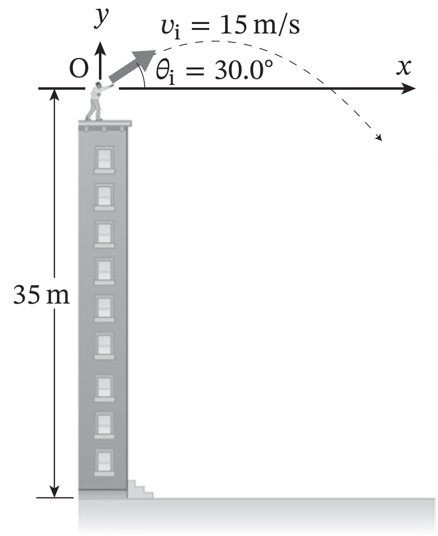
You stop the motion of Block A. Then, even after you release it, it does not start moving again.

(5) Describe what is known about the value of μ_s.

[The exam questions continue on the next page.]

[Part C] Projectile Motion (14 points)

In this part, use $g = 10\,\mathrm{m/s^2}$ as the standard acceleration of gravity (instead of $9.8\,\mathrm{m/s^2}$) to simplify the calculation.



A stone is thrown from the top of a building upward at an angle $\theta_\mathrm{i} = 30.0°$ to the horizontal with an initial speed $v_\mathrm{i} = 15\,\mathrm{m/s}$. The height from which the stone is thrown is 35 m above the ground.

As shown in the right figure, we take the point from which the stone is thrown as the origin O, and define x-axis rightward and y-axis upward. Answer the following questions with neglecting air resistance.

First, assume there is no wind.

(1) How long does it take the stone to reach its highest point?

(2) How long does it take the stone to reach the ground?

(3) What is the velocity of the stone just before it strikes the ground?

Now, assume a horizontal wind is blowing rightward (positive-x direction) and it causes the stone to have a horizontal acceleration $a_x\vec{e_x}$. A stone is thrown at $t = t_0$ and it reaches the ground at $t = t_1$.

(4) Let $a_x = 0.50\,\mathrm{m/s^2}$. Find $t_1 - t_0$.

(5) As we have defined x- and y-axes (as in the figure), we can describe the position of the stone at the time t, where $t_0 \le t \le t_1$, by a position vector $\vec{r}(t)$.

(a) Describe $\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{r}(t)$ by using *all of* the following symbols: $\vec{e_x}$, $\vec{e_y}$, a_x, and g.

(b) Describe $\frac{\mathrm{d}}{\mathrm{d}t}\vec{r}(t)$ by using *all of* the following symbols: $\vec{e_x}$, $\vec{e_y}$, a_x, g, v_i, θ_i, t_0, and t.

[The exam questions continue on the next page.]

Answer either **[Part D]** or **[Part E]**. [Hint: Part E is a bit advanced and difficult.]

Please specify clearly which part you answer, especially if your answer sheet contains answers for both parts.

[Part D] Learning at University (12 points)

(1) When studying as an adult, it is essential to set clear goals and keep them in mind. Here are the "Students' Goals" given in this course's syllabus. Fill the blanks **(A)–(G)** by suitable words.

- I know how to study at the university. I have "my way of learning" customized for my personality and ___**(A)**___.
- I know how to study efficiently, ___**(B)**___, and individually based on English textbooks.
- I can perfectly handle quantities with ___**(C)**___ without confusing ___**(D)**___ and ___**(E)**___.
- I am familiar with ___**(F)**___, integrals, and ___**(G)**___, and can use them to analyze three-dimensional motions.
- I can explain "energy", "work", "potential", "momentum", and relations among them.
- I can find and describe forces in a system with multiple rigid objects.

[Hint: (A), (C), and (G) should contain two words and others should be a word, but your answer does not need to be exactly the same as in the syllabus. Accepted if it fits.]

(2) Almost all first-year university students encounter learning challenges due to the differences in the learning environment compared to high school. How about you? Write at least three difficulties **you** encountered during this two months. In addition, provide some ideas to overcome (resolve) those difficulties.

[Part E] Calculus (12 points)

Calculate the following derivatives and antiderivatives (indefinite integrals), where n is an integer.

(1) $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{\sin x^2}{\cos x}$

(2) $\dfrac{\mathrm{d}}{\mathrm{d}x}\cosh^2 x$

(3) $\dfrac{\mathrm{d}}{\mathrm{d}x}\log_{112}\left(2023x^{1025}\right)$

(4) $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{\ln x+\sqrt{x}}{\sin(x^2)+\mathrm{e}^{-x}}$

(5) $\displaystyle\int 3^x\,\mathrm{d}x$

(6) $\displaystyle\int \cos(3x+2)\,\mathrm{d}x$

(7) $\displaystyle\int x^n\,\mathrm{d}x$

[The exam questions continue on the next page.]

[Part F] Extra Problem (unlimited points)

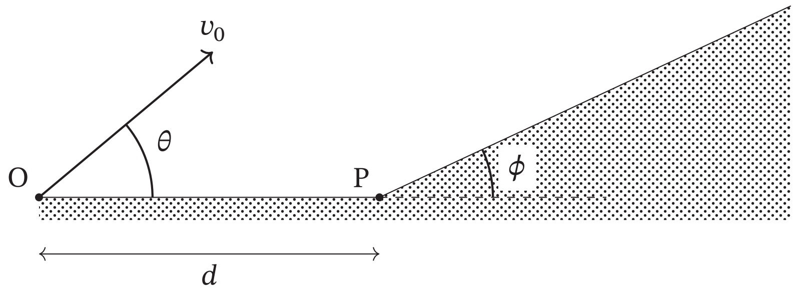
> This is an extra challenging problem, but make an attempt to write something for a partial mark. You are still a student, so no penalty for wrong answers!

Imagine you are in a golf competition, in military service, a protester throwing an egg, or whatever you like. In any case, you are on the Earth and you are going to shoot a golf ball (or fire a cannon).

You are going to launch a projectile from Point O. Its initial speed is fixed at v_0, but you can vary the launch angle θ from 0° to 90°, i.e., $0° < \theta \le 90°$. As shown in the figure, the terrain in the launching direction is flat (horizontal) up to Point P, which is at a distance d from Point O. However, beyond Point P, there is a very long uphill slope with an angle ϕ.

Discuss physics based on the following information and approximations.

- Ignore air resistance.
- The mass of the projectile is m.
- Because v_0 is much smaller than 1 km/s, ignore the fact that the Earth is round and use the standard gravitational acceleration g throughout your answer.
- Assume the slope goes on endlessly, where $0° < \phi < 90°$.
- No information is available on the value of d. It may be shorter than 1 cm, or longer than 1 km.



[Hint: You may discuss anything. For example, Sho wants to know the angle θ that allows the projectile to go farthest, and the angle that maximizes the flight time of the projectile. Are they equal, or different?]

[This is the end of the exam questions.]