LABORATORY STUDIES OF PARTICLE DISPERSION IN TWO-DIMENSIONAL TURBULENCE

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Abstract- *The absolute dispersion, Lagrangian velocity correlations, eddy diffusivity, structure function, and energy spectra characteristics of a quasi-two-dimensional turbulent flows are investigated in this study using a combined experimental-numerical technique. Experimented results, especially those for the absolute dispersion and eddy diffusivity and their asymptotic laws in the initial dispersion regime, correlate well with the classical diffusion-theory. On the other hand, the calculated Lagrangian statistics at large times are somewhat inconsistent with the predicted results due to nonstationarity of turbulence. This discrepancy can be substantially reduced if the dispersion analyses are based on the mean particles energy at t=0 and t=T (T is the Lagrangian integral time scale), respectively, for small and large times regimes. Furthermore, the slope of the Lagrangian energy spectrum beyond the cut-off frequency is close to -2, which is similar to previous observational results of Lagrangian velocity spectra as deduced from surface or subsurface drifters in the ocean.*

Keywords : Turbulent diffusion, Two-dimensional turbulence, Lagrangian statistics, Eddy diffusivity

INTRODUCTION

The use of freely drifting particles to measure or observe the flow field has become increasingly relevant in many branches of fluid mechanics, e.g., physical oceanography, atmospheric science and hydrodynamics. Compared with the typical Eulerian descriptions of flows by the fixed-point measuring device, the current-following floats or drifters provide a Lagrangian view of flows and have led to many discoveries in areas such as mixing and diffusion in turbulent fields. Taylor's [1] classical analysis of particle motion in statistically homogeneous and stationary turbulence provides a framework for describing mean eddy fluxes. The problem of diffusion in a field of homogeneous turbulence was later considered and its asymptotic equations were derived by Batchelor [2]. More recently, Babiano et al. [3] theoretically and numerically investigated the single-particle dispersion, Lagrangian structure functions and Lagrangian energy spectra characteristic of two-dimensional incompressible turbulent flows. Their works confirmed the classical asymptotic estimates of single-particle dispersion at small and large times, as provided by Taylor [1] and Batchelor [2]. On the other hand, Freeland et al. [4] first studied the Lagrangian properties of neutrally buoyant floats drifting at 1500m depth in the ocean. They found that the classical diffusion theory for homogeneous turbulence was incapable of accurately describing the dispersion of their floats ; in addition, the nonstationarity of the statistics was attributed as the reason for this discrepancy.

These studies, among others, have motivated the present investigation of single-particle dispersion, the Lagrangian structure function, and the Lagrangian energy spectrum in a two-dimensional turbulence using a combined experimental-numerical technique devised earlier by Tseng and Maxworthy [5]. The turbulent field generated by our laboratory apparatus is well-controlled and serves as a good test of Taylor's [1] hypothesis. Meanwhile, the calculation method adopted

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in this study provides a valuable description of float trajectories. A comparison is then made of results obtained in this study with previous field measurements of Freeland et al. [4] and other studies, and with the theoretical and numerical results of Babiano et al. [3].

EXPERIMENTS

The experiments were conducted in a square tank (Figure 1) of dimensions $240cm \times 240cm \times 25cm$. The tank was filled with a two-layer system consisting chiefly of an upper layer of fresh water lying over salt water of density $1.040 g/cm^3$. Both layers were of the same depth of 6.5 cm and were filtered overnight before introducing into the tank. A thin layer of salt-saturated water, $1cm$ deep, was introduced beneath this two-layer fluid to reduce the interaction of vortices with the bottom of the tank. A large number of polystyrene particles (a few thousands), which are commercially available with a density of about $1.04\ g/cm^3$ and of dimension $3.5mm \times 3.0mm \times 2.0mm$, were placed in the fresh-salt water interface to mark the flow field. Turbulence was created by towing a grid of vertical bars $3cm$ wide, set on $15cm$ centers through the depth of the fluid from one end to the other end of the tank and then back to the initial position to eliminate any mean motion. The grid was towed at a constant speed of $5cm/sec$, which was rapid enough to produce a large intensity of turbulence, yet slow enough to prevent the generation of unwanted internal waves that might introduce distortions. A short transient period of three-dimensional turbulent motion was first generated. This three-dimensional turbulent motion was suppressed vertically by the stratification and it quickly became quasi-two-dimensional. In this approach, a homogeneous, quasi-two-dimensional turbulent flow was generated at the sharp density interface of the two-layer system. The evolution of this quasi-two dimensional turbulent flow was visualized by photographing the neutrally buoyant particles over a period of about 20 minutes from above the tank with time exposures between 4 and 13 seconds. The particle streaks were then digitized, and the resulting velocities interpolated onto a regular 64×64 grid. Ten different particle streak photographs were digitized. These photographs were taken at about 1 to 20 minutes after the start of particle motion. The digitized results were used for further analysis of particle trajectories and Lagrangian statistics, as is described in the following sections.

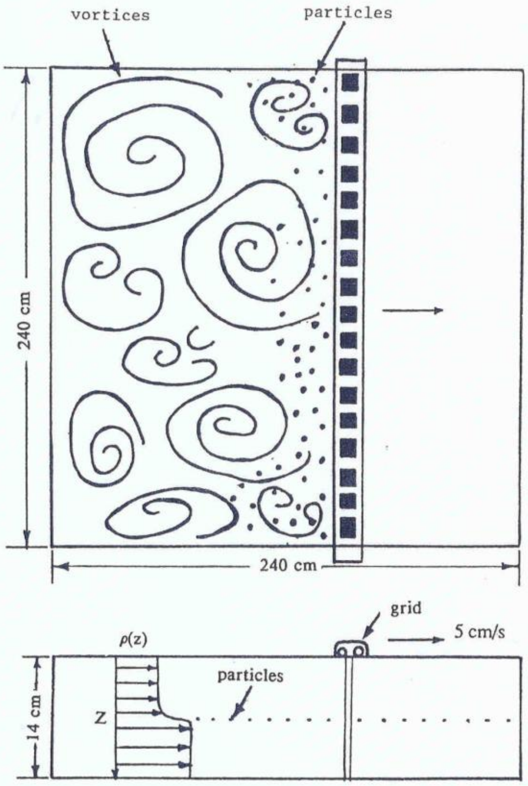


Fig. 1 Apparatus of experiments.

EDDY FIELDS AND PARTICLE TRAJECTORIES

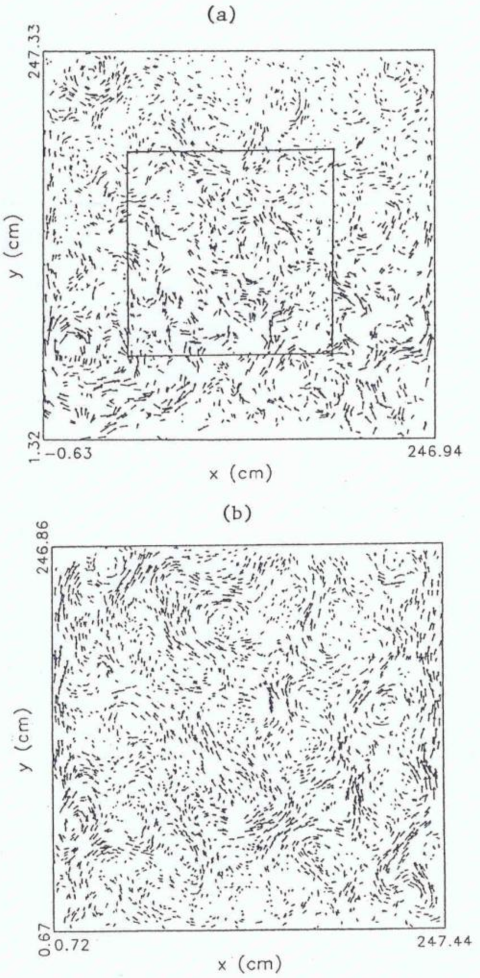
Four of the ten streak photographs digitized in this study are shown in Figure 2 to illustrate the evolution of the two-dimensional turbulence. These four photographs represent the digitized velocity fields at 103, 149. 310, and 610 seconds, respectively, after the start of grid motion. The growth of the large number of vortices to larger scales by vortex merging as time progressed can be clearly observed. Previous results on the vortex dynamics and front geometry in an experimentally generated two-dimensional turbulence have been reported by Maxworthy et al. [6] and Tseng and Maxworthy [5]. Our main interest here is, however, on the Lagrangian properties of these eddy fields in two-dimensional turbulence.

The trajectories of a large number of neutrally buoyant Lagrangian particles in a two-dimensional turbulent flow can be determined by a combined experimental-numerical method. This method, which has been used in another study (Tseng and Maxworthy [5]), to compute the evolution of a two-dimensional turbulent/nonturbulent front, was found by them to have reasonably good accuracy of the front location.

The procedures of this method are summarized here. First a square patch of 10×10 "imaginary" particles was inserted into the center region of the initial velocity field (Figure 2a). This initial velocity field was assumed to be frozen for a certain time interval, generally that between successive frames. Next, the particles were advanced by the frozen velocity field to their new positions using a first-order forward scheme with a time step τ,

$$x(t+\tau) = x(t) + \tau V(x(t)) \qquad (1)$$

where $x(t)$ and $x(t+\tau)$ are the positions of a certain particle at times t and $t+\tau$, respectively, and the velocity V at $x(t)$ was evaluated by linear interpolation within the relevant mesh. The time step τ was selected to be 3 seconds, which is smaller than the exposure time of the corresponding streak photograph. This process was repeated several times by substituting the updated particle positions into each velocity field at each time step. This approach was taken because, at the time, we were not able to follow so many real particles from frame to frame in a video sequence.



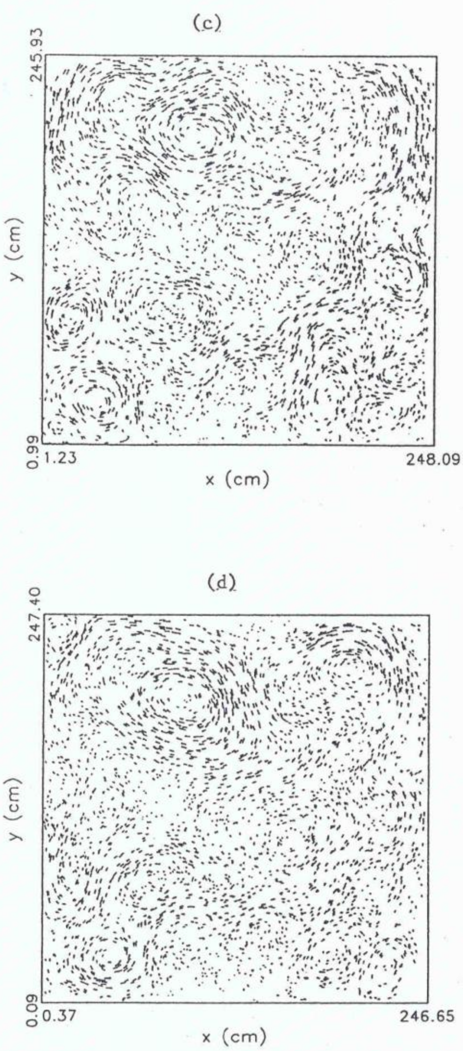


Fig.2 Digitized velocity fields at (a) 103, (b) 149, (c) 310, and (d) 610 seconds after the start of grid motion. The square box shown in (a) indicates the location within which imaginary particles were launched initially (see text for detail)

The number of imaginary particles was increased to 1000 for the sake of eliminating the discontinuities in slope due to the finite number of realizations as well as enhancing the computational accuracy of the Lagrangian statistics. These drifters were divided into 10 ensembles, each ensemble was launched at various times during the course of the experiment. The number of particles released and their corresponding time of release are summarized in Table 1. Note that the total drift time of each particle was maintained constant (600 seconds).

Table 1. Primary Statistics

Ensemble e	Number of particles	Launching times after the start of grid motion (sec)	$\bar{u}$	$\bar{v}$ (in cm/s)	u_{rms}	v_{rms}
1	100	124	-0.056	0.011	0.150	0.166
2	100	133	-0.056	0.014	0.149	0.163
3	100	139	-0.061	-0.002	0.138	0.156
4	100	148	-0.064	-0.001	0.134	0.152
5	100	154	-0.063	-0.005	0.124	0.144
6	100	163	-0.066	-0.007	0.120	0.141
7	100	169	-0.068	-0.012	0.114	0.131
8	100	187	-0.066	-0.015	0.114	0.130
9	100	205	-0.050	-0.032	0.109	0.104
10	100	265	-0.035	-0.032	0.102	0.096
		mean	-0.059	-0.008	0.125	0.138

Let u, v be the components of the velocities in the directions perpendicular and parallel to the direction of grid movements, respectively. The mean velocities $\bar{u}, \bar{v}$, and the standard deviation u_{rms}, v_{rms}, due to the eddy field for each ensemble, were computed and those results are listed in Table 1. The mean velocities in the v component are generally more insignificant than the r.m.s. velocities of the eddy field. However, it is not quite the same situation in the u component. One important result inferred from Table 1 is that the eddy velocities are closely isotropic ($u_{rms} \doteq v_{rms}$).

LAGRANGIAN STATISTICS

Lagrangian statistics such as the absolute dispersion, the Lagrangian structure function, the velocity correlation coefficient, the integral time scale, eddy diffusivities, and energy spectra can be computed and analyzed from the large amount of data, which consisted of time series of position and velocity of 1000 drifters with a 3-sec interval and 200 time steps. As mentioned earlier, Lagrangian dispersion analyses are usually based on the classical theory of Taylor [1] and Batchelor [2]. Under the assumptions of a homogeneous, stationary, zero-mean, two-dimensional turbulent field, a number of asymptotic laws have been found for the absolute dispersion of fluid parcels from their initial position. The domain of validity of the classical asymptotic estimates was further extended by Babiano et al. [3]. Some of those details are presented below.

Consider the time evolution of the dispersion of a number of fluid particles from their initial positions in a homogeneous and stationary turbulent field. The absolute dispersion $D(t)$ can be defined as

$$D(t) = \langle (\int_0^t V(a,\tau)d\tau)^2 \rangle \tag{2}$$

where $V(a,t)$ is the Lagrangian description of particle velocity in terms of its initial position a as function of time t, the symbol $< \cdot >$ denotes the ensemble average over many independent drifters. The eddy diffusivity $K(t)$ is then defined as

$$K(t) = \frac{1}{2}\frac{d}{dt}D(t) \tag{3}$$

The Lagrangian structure function $S(t)$ is defined as

$$S(t) = \frac{1}{2} < |V(a,0) - V(a,t)|^2 > \tag{4}$$

and the Lagrangian velocity correlation coefficient $R(t)$ is

$$R(t) = \frac{< V(a,0)V(a,t) >}{< |V(a,0)|^2 >} \tag{5}$$

while the Lagrangian integral time scale T is

$$T = \int_0^\infty R(t)dt \tag{6}$$

The Lagrangian integral time scale is generally regarded as a measure of the time scale during which a particle "remembers" its initial condition. After the time T the particle velocities $V(a,t)$ become statistically independent or uncorrelated with themselves.

Taylor [1] first showed that an important relationship occurs among the eddy diffusivity, absolute dispersion, and velocity correlation coeffficient. These relationships are

$$K(t) = \frac{1}{2}\frac{d}{dt}D(t) = 2E\int_0^t R(\tau)d\tau \tag{7}$$

where E is the mean initial energy of the particles, i.e.,

$$E = \frac{1}{2} < |V(a,0)|^2 > \tag{8}$$

We thus obtain

$$D(t) = 4E\int_0^t (t-\tau)R(\tau)d\tau \tag{9}$$

Because $R(\tau) \to 1$ at $\tau \to 0$ and $R(\tau) \to 0$ at τ very large, eqs. (7) and (9) independently approach two limits, that is,

$$\left.\begin{array}{l} D(t) = 2Et^2 \\ K(t) = 2Et \end{array}\right\} , \text{ for } t << T \tag{10a}$$

(i.e. for the initial dispersion phase)

$$\left.\begin{array}{l} D(t) = 4ETt \\ K(t) = 2ET \end{array}\right\} , \text{ for } t >> T \tag{10b}$$

(i.e. for the random walk phase)

Note that for a stationary turbulent field, the mean energy of the flow approachs the mean initial energy of the drifters. The mean energy of 1000 drifters, $E(t)$, is plotted in Figure 3 as a function of time. This figure indicates that the mean energy of drifters decreased continuously from $0.08cm^2/sec^2$ in the beginning to

$0.05 cm^2/sec^2$ at $t = T$ (the value of the Lagrangian integral time scale T is obtained in the next section), and then decreased to almost $0.006\ cm^2/sec^2$ at $t = 600 sec$. As a result of this slowly decaying field, the classical diffusion theory must be modified. A simple method is proposed in this study to calculate the Lagrangian statistics : In the initial dispersion phase (eq. 10a), the mean initial energy at $t = 0$, denoted by E_0, was used to calculate $D(t)$ and $K(t)$. In the random walk phase at $t >> T$ (eq. 10b), the mean energy at $t = T$, denoted by E_T, was used instead.

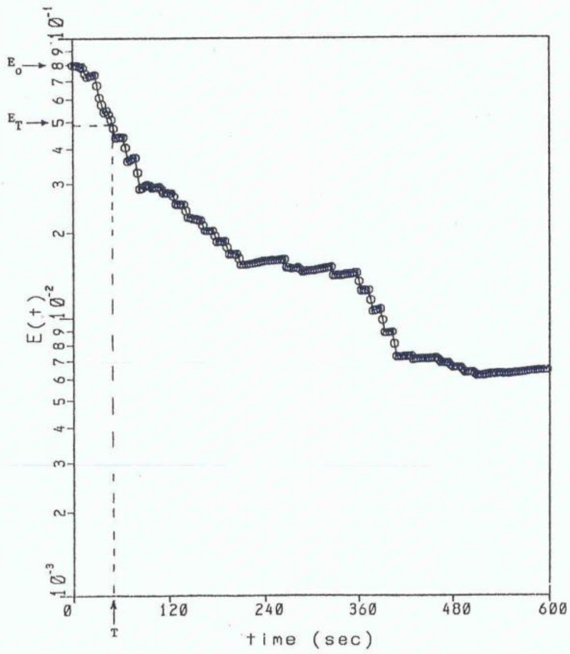


Fig.3 Mean energy of particles as a function of time. The mean initial energy at t=0 and the mean energy at t=T are also marked.

The Lagrangian velocity correlations, $R(t)$, obtained from eq. (5), is shown in Figure 4. The Lagrangian integral time scale, as calculated from eq. (6), was found to be 45.6 seconds for this experiment. For times less than the integral time scale, the particles are moving with a motion that is highly correlated with their initial velocity, and the value of $R(t)$ is generally greater than about 0.5. For times much greater than the integral time scale, $R(t)$ has the form of oscillating sidelobes with the value ranging between 0.1 and -0.1, thereby indicating that the particles are moving in the random walk regime.

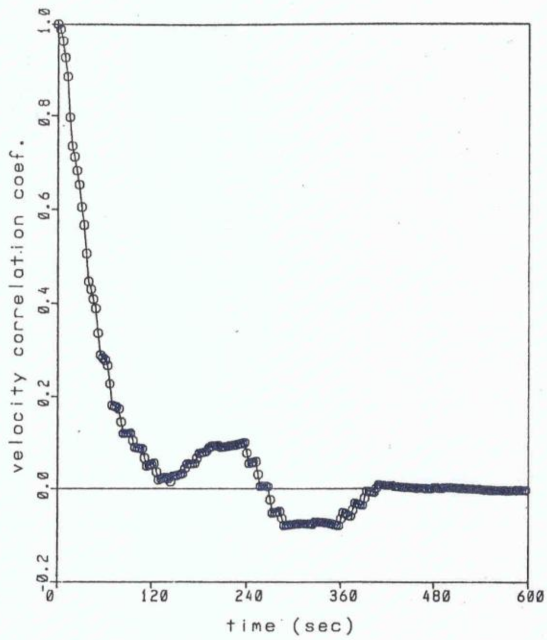


Fig.4 Lagrangian velocity correlation coefficient as a function of time.

The Lagrangian structure function $S(t)$, computed from eq.(4), is displayed in Figure 5. In the ideal case the structure function should increase rapidly to a saturation level, given by $S(t) = 2E_0$, and then remains constant. This theoretical constant level $2E_0$ is also plotted in Figure 5 and correlates sufficiently with our experimental results for $t >> T$. Babiano et al. [3], in their theoretical study, concluded that the Lagrangian structure function saturated at a t^2 dependency if the slope n of Lagrangian energy spectra was steeper than -3, and the structure function followed a $t^{|n|-1}$ dependency if the slope n was between -1 and -3. This finding is, however, inconsistent with our results of Lagrangian structure function for $t << T$.

The absolute dispersion $D(t)$ is plotted in Figure 6 as a function of time. The asymptotic law of initial disperison $D(t) = 2E_0t^2$ is obviously verified to extend from small times to time scale of the order of τ. At $t \geq 2T, D(t)$ is seen to have a rather abrupt shift of slope toward a linear dependency in time. The asymptotic linear behavior of absolute dispersion at larger times, $D(t) = 4E_TTt$, was found to be consistent reasonably well with our experimental results. As mentioned earlier, nonstationarity of the statistics was attributed by Freeland et al. [4] as the reason for the discrepancy between their results of float dispersion and Taylor's prediction. The present study further confirmed the importance of the stationarity in the Lagrangian disperison studied, and has proposed a simple connection scheme for the decaying turbulence field.

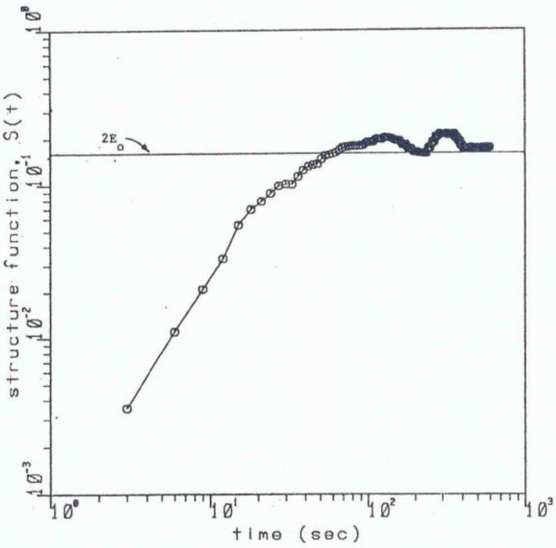


Fig.5 Lagrangian structure function as a function of time.

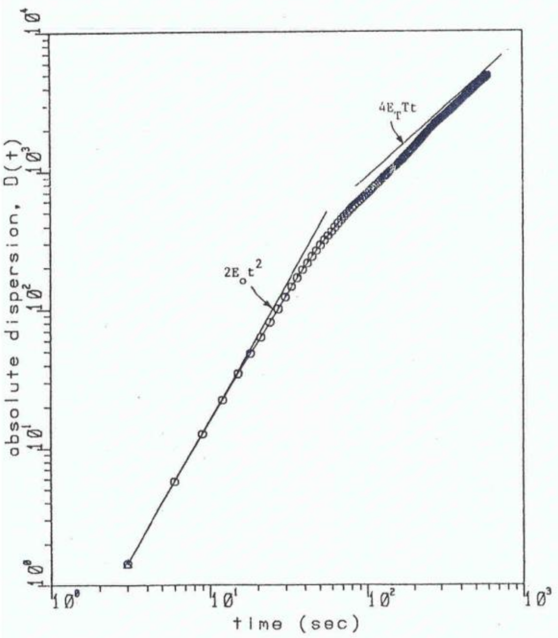


Fig.6 Absolute dispersion versus time. The two straight lines represent the theoretical values predicted by Eq. (10a) and (10b).

The eddy diffusivity $K(t)$, obtained from time differentiation of the absolute dispersion $D(t)$, is shown in Figure 7 as a function of time. At brief time intervals in the initial dispersion regime, $K(t)$, is seen to have a linear dependency in time which is in excellent agreement with that predicted by the asymptotic law, $K(t) = 2E_0t$. At longer time intervals in the random walk regime, the constant eddy diffusivity $2E_TT$ predicted by the asymptotic law is also compatible with the experimental results.

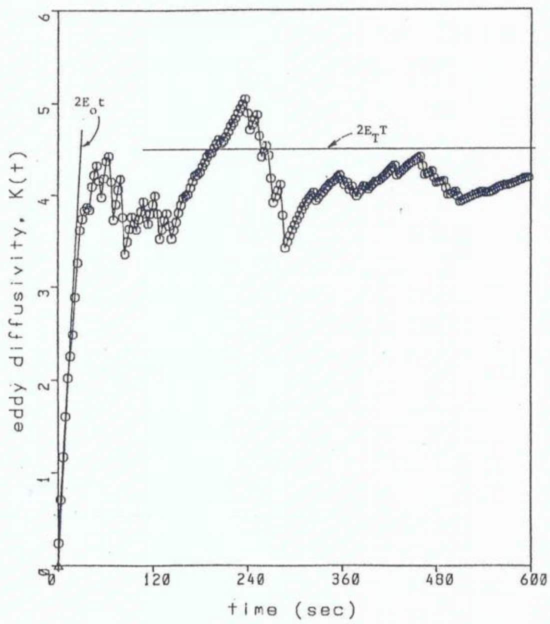


Fig.7 Eddy diffusivity as a function of time. The two straight lines are from Eq. (10a) and (10b).

The Lagrangian energy spectra are shown in a log-log plot in Figure 8. The spectra consist of a low-frequency plateau which extends up to the cut-off frequency given by the frequency of the energy-containing eddies. At higher frequencies, the spectra have a slope close to -2. Freeland et al. [4] obtained slopes of Lagrangian energy spectra in $1500m$ depth like -4. On the other hand, Colin de Verdiere [7] obtained a well defined -2 spectra slope of Lagrangian velocity spectra from surface drifters in the eastern North Atlantic. Krauss and Boning [8] also estimated the Lagrangian spectra from subsurface drifters (at $100m$ depth) in the North Atlantic. Their spectra fall off predominantly acccording to -2, but in some cases are better described by a slope of -3. Therefore, the value of the spectra slope determined from our combined experimental-numerical experiments apparently correlates better with the previous ocean measurements as deduced from surface or subsurface drifters. This also suggests the similarity of eddy fields between the present laboratory experiments and the oceanic surface flows.

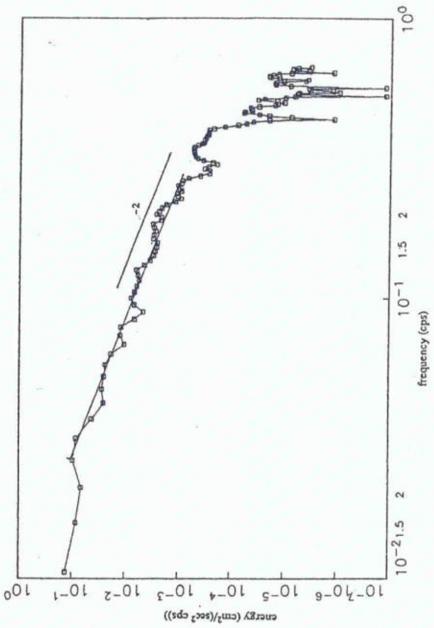


Fig.8 Lagrangian energy spectra for the entire 600 seconds. The straight line with -2 slope is also indicated

CONCLUSIONS

Both the Lagrangian behavior and statistics of drifters in a quasi-two-dimensional turbulent field have been experimentally and numerically investigated in this work. Our results on the single particle dispersion and Lagrangian eddy diffusivity, at small time intervals in the initial dispersion phase, are quite consistent with the prediction of classical diffusion theory (Taylor, [1]). Due to the slow decay of the turbulence, however, the calculated Lagrangian statistics at large time intervals, in the random walk regime, are somewhat smaller than the predicted results. This discrepancy can be substantially reduced by substituting the mean particles energy at the Lagrangian integral time scale into the asymptotic laws in the random walk regime. Finally, beyond the cut-off frequency of the eddies the slope of Lagrangian energy spectra follows a -2 power law.

This is in accordance with the observational results of Lagrangian velocity spectra as deduced from surface or subsurface drifters in the oceans. The results obtained in this study will be helpful in describing the Lagrangian properties of turbulent flows, especially on the diffusion and mixing of pollutants in the coastal environmentss where the turbulent fields may not be fully stationary.

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質點在二維紊流場中的擴散特性

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摘　要

本文是以一結合實驗與數值計算的方法研究質點在一近似二維紊流場中的拉氏擴散特性。研究結果發現在初始擴散階段時，所求得的絕對擴散距離以及渦流擴散係數值與古典的擴散理論值相當吻合。另一方面，在較長時間之後，所求得的擴散特性則與理論結果有差異，這顯然是由於紊流場的強度隨著時間而減弱（或不穩定性）的緣故。如果計算過程予以適當修正，亦即根據質點在 t＝0 與 t＝T（T 是拉氏積分時間尺度）的平均動能分別來推算初始階段與長時間階段的擴散特性，則上述差異將可大幅改善。最後，本研究所求得的能譜斜率大約爲 -2 ，這與過去海洋表層或次表層浮標飄流觀測所得之拉氏能譜斜率頗爲近似。

關鍵詞：擾流擴散，二維紊流，拉氏統計特性，渦流擴散係數
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