Capital Mobility, Interest Rate Rules, and Equilibrium Indeterminacy in a Small Open Economy

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Abstract

We develop a monetary model of the small open economy operating under flexible exchange rates and either with or without international capital mobility. Money is introduced into this economy through the channel of shopping-time technology. We find that in a small open economy with capital immobility an active interest rate rule yields determinacy of equilibrium while a passive rule shows indeterminacy. In a small open economy with capital mobility, the equilibrium displays determinacy regardless of the type of interest-rate feedback rules.

Keywords: Capital mobility; Interest rate rules; Indeterminacy; Small open economy

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1. Introduction

It is well known in the literature on interest rate rules that the active interest-rate rule, the policy by which the monetary authority raises nominal interest rates by more than the rise in inflation, will stabilize the economy by ensuring a unique equilibrium. This has sprouted a growing body of research to investigate its validity. Among these, Benhabib et al. (2001) employ a monetary endowment economy with money entering preferences and/or production technology to show that interest rate rules cause indeterminacy in equilibrium.

Yet all these studies are restricted to the analysis of closed economies. As increasing integration has prevailed in international goods and asset markets in recent years, domestic macroeconomic performance has become more sensitive to foreign shocks, especially under the ebbs and flows of foreign capitals. This fact naturally inspires us to explore the relationship between the interest-rate feedback rule and macroeconomic stability from the perspective of an open-economy framework. This paper specifically makes a new attempt to assess the role of international capital mobility in impacting the relationship between interest-rate feedback rules and macroeconomic stability.

The role of international capital mobility has long attracted much attention by international macroeconomists. Fleming (1962) and Mundell (1963) assess how capital mobility affects the effectiveness of macroeconomic policies under alternative exchange rates. Frenkel and Rodríguez (1982) examine the effect of capital mobility on exchange rate dynamics. Obstfeld (1982) further notes that, when the economy is initially in external balance, the current-account effect of terms-of-trade shocks is different under perfect capital mobility and imperfect mobility. Turnovsky (1997) later emphasizes that a temporary policy has a permanent effect under perfect capital mobility while a temporary policy only has a temporary effect under imperfect capital mobility. In view of these developments, it is a worthwhile task for us to study how international capital mobility affects the link between interest-rate feedback rules and macroeconomic stability in

1 The related literature includes, for example, Leeper (1991), Woodford (1995), Taylor (1999), Clarida et al. (2000), and Carlstrom and Fuerst (2001). Furthermore, extended discussions of interest rate rules in both the history of economic thought and modern analysis of theory can be found in Woodford (2003).
an open economy. To address this link, we extend the framework of Kimbrough (1986) and Ljungqvist and Sargent (2000) to a continuous-time setting of a small open economy operating under flexible exchange rates and either without capital mobility or with capital mobility. We find that, in a small open economy without capital mobility an active interest rate rule yields a determinate, and a passive rule, indeterminate equilibrium. By contrast, in a small open economy with capital mobility, the equilibrium displays determinacy regardless of interest rate rules.

2. A small open economy without capital mobility

Assume that the country is a small open economy and operates under flexible exchange rates. We first consider that this economy has no access to the world capital market. This implies no international capital mobility. The economy produces a single traded good whose price in terms of foreign currency is given on the world market and is normalized to be unity. That is,

\[ P = E P_f, \]  

where \( P \) is the price level of traded goods in home currency, \( E \) is the exchange rate (defined as the home currency price of foreign currency), and \( P_f \) is foreign prices of traded good (\( P_f = 1 \)).

The domestic country consists of an infinitely-lived representative agent and a government. The agent is endowed with a constant output \( y \) each instant of time and may hold two assets (domestic money and bonds). The agent’s objective is to maximize the following lifetime utility

\[ \int_{0}^{\infty} u(c, l) e^{-\rho t} dt, \]  

subject to

\[ \dot{a} = y + (R - \pi)a - Rm - c - \tau, \]  

\[ l + s = 1, \]  

where \( c \) = consumption, \( l \) = leisure, \( \rho \) = a constant rate of time preference, \( a \equiv m + b \) = total real wealth, \( m \equiv M/P \) = real money holdings, \( b \equiv B/P \) = real bond holdings, \( M \) = nominal money holdings, \( B \) = nominal bond holdings, \( R \) = nominal interest rates, \( \pi \equiv \dot{P}/P = \dot{E}/E \) = rate of inflation (rate of exchange depreciation), \( \tau \) = lump-sum taxes, and \( s \) = shopping-time technology.
Equation (2) says that the representative agent maximizes the discounted lifetime utility with perfect foresight. The instantaneous utility function $u$ satisfies $u_c > 0$, $u_t > 0$, $u_{cc} < 0$, $u_{tt} < 0$, $u_{ct} > 0$, and $u_{cc}u_{tt} - u_{ct}^2 > 0$. Equation (3) is the agent’s budget constraint. Equation (4) is the constraint of time allocation. The time endowment is normalized to be unity and the agent allocates his or her time endowment to leisure and time spent transacting.

We follow Kimbrough (1986) and Ljungqvist and Sargent (2000) to motivate the agent for holding money in the economy by a transaction technology with shopping time. It describes that the amount of shopping time $s$ required for purchasing each unit of consumption depends on the ratio of real money holdings to consumption. Specifically, the shopping-time technology is

$$s = g(m/c)c; \quad g' < 0, \quad g^* \geq 0.$$  

Equation (5) states that the shopping time is an increasing function of consumption and a decreasing function of real balances.

We now substitute equation (5) into equation (4) and let $\lambda$ be the co-state variable and $\gamma$ the multiplier of the current value Hamiltonian associated with (3) and (4). The necessary optimum conditions for the representative agent are

$$u_c(c, l) = \lambda + \gamma[g(m/c) - g'(m/c)m/c],$$  

$$u_t(c, l) = \gamma,$$  

$$\lambda R = -\gamma g'(m/c),$$  

$$\dot{\lambda} = \lambda(\rho + \pi - R),$$

together with equations (3) and (4), and the transversality condition of $a$,

$$\lim_{t \to \infty} \lambda ae^{-\rho t} = 0.$$  

Following Leeper (1991), Benhabib et al. (2001), and Carlstrom and Fuerst (2001), we assume that the monetary authority adopts an interest-rate feedback rule as

$$R = \psi(\pi), \quad \psi' > 0.$$  

From equation (8), monetary policy is active at an inflation rate $\pi$ if $\psi' > 1$, and is passive if $\psi' < 1$. We further assume that at all times the government incurs no expenditure and finances its
deficit by raising seigniorage and issuing bonds. Accordingly, the government’s budget constraint in real value can be stated as

\[ \hat{a} = (R - \pi)a - Rm - \tau. \]  

(9)

With equations (7) and (9), we mainly focus on the Ricardian fiscal policy.

Equations (3) and (9) give the balance-of-payments equilibrium:

\[ 0 = y - c. \]  

(10)

With no international capital mobility, the balance of payments equals the balance of trade. Because there exists only one traded commodity in the world, the balance of trade is the excess of domestic production over domestic absorption. Under flexible exchange rates, the balance of trade is in equilibrium at all times. As a consequence, the perfect-foresight equilibrium of a small open economy may be expressed by equations (6a)-(6d), (4), (5), and (7)-(10).

As equations (6a)-(6c), (4), (5), (8), and (10) hold at all times, manipulating these equations gives

\[ u_c(y, 1 - g(m/y) y) = \lambda + u_i(y, 1 - g(m/y) y)[g(m/y) - g'(m/y)m/y], \]  

(11a)

\[ \lambda \psi(\pi) = -u_i(y, 1 - g(m/y) y)g'(m/y). \]  

(11b)

From equations (11a) and (11b), we further have

\[ m = m(\lambda), \]  

(12a)

\[ \pi = \pi(\lambda), \]  

(12b)

where \( m_\lambda = 1/\{-g'[u_{cl} - u_{li}(g - g' \cdot \frac{m}{y})] + u_i g'' \cdot \frac{m}{y^2}\} > 0, \)

\[ \pi_\lambda = \{(g')^2 u_{li} - \frac{u_{li}}{y} \cdot g'\} m_\lambda - \psi \cdot \frac{1}{\lambda \psi'} < 0. \]

Differentiating equation (12b) with respect to time and using equations (6d) and (8), we have

\[ \hat{\pi} = \lambda \pi_\lambda [\rho + \pi - \psi(\pi)]. \]  

(13)

From equation (13) with \( \hat{\pi} = 0, \) the steady-state value of exchange rate depreciation \( \pi^* \) is

\[ \rho = \psi(\pi^*) - \pi^* \equiv r, \]  

(14)

where \( r \) denotes the real interest rate. Linearizing equation (13) at \( \pi^* \) yields

\[ \hat{\pi} = \phi(\pi - \pi^*), \]  

(15)
where $\phi = \lambda \pi [1 - \psi'(\pi^*)]$. Because $\phi$ is the characteristic root of equation (15), the equilibrium at $\pi^*$ is determinate or indeterminate depending on the sign of $\phi$. Given $\pi_\lambda < 0$ in equation (12b), the sign of $\phi$ hinges on the type of monetary policy. Since the rate of exchange depreciation is a non-predetermined variable, the equilibrium shows determinacy if the monetary policy is active ($\psi' > 1$) while the equilibrium is under indeterminacy if the monetary policy is passive ($\psi' < 1$). Based on the above analysis, we come to the following proposition:

**Proposition 1.** In a small open economy without capital mobility, the active interest rate rule yields a unique perfect-foresight equilibrium, while the equilibrium displays indeterminacy under a passive monetary policy rule.

The intuition for our result is the following. Assume that initially the inflation rate (exchange rate depreciation) is above its steady-state level. If the monetary policy is active, then a rise in the inflation rate brings an increase in the real interest rate. This in turn induces the shadow price $\lambda$ to decrease, because $\dot{\lambda}/\lambda = \rho + \pi - R < 0$ is true. A lower shadow price then raises desired consumption and lowers the marginal utility of consumption. Since the balance of trade must be in equilibrium under flexible exchange rates, consumption equals endowment at all times. In order to optimally follow the declining path of the marginal utility of consumption, agents thus must cut leisure time as a result of $u_{cl} > 0$. However, less leisure time means more time spent transacting, which thereby lowers real money holdings and further raises the inflation rate. This obviously drives the inflation rate away from its value at the steady state. If, on the other hand, the monetary policy is passive, then a rise in the inflation rate brings about a fall in the real interest rate. A fall in the real interest rate will raise the shadow price and then the marginal utility of consumption. To match the higher marginal utility so as to keep the consumption constant, agents must raise leisure time. However, more leisure means less shopping time, which in turn raises the need for real balances and lowers the inflation rate. Therefore, this trajectory is consistent with an equilibrium in which inflation converges to its stationary value.
3. A small open economy with capital mobility

We now relax the assumption of no international capital mobility and assume that the home economy faces an imperfect world capital market as proposed by Obstfeld (1982) and Turnovsky (1997). In this regard, the domestic resident may hold an internationally-traded bond denominated in foreign currency in addition to domestic money and bonds. Furthermore, the cost (benefit) of borrowing (lending) faced by the domestic resident is an increasing (decreasing) function of his or her indebtedness (creditworthiness) to the rest of the world. As a consequence, we assume that

\[ R_f = R_f(b_f), R'_f \equiv dR_f/db_f < 0, \]  

where \( R_f \) represents foreign nominal interest rates and \( b_f \) is the stock of foreign bonds denominated in foreign currency.

With capital mobility, the agent’s budget constraint is revised as follows\(^2\)

\[ \dot{a} = y + (R - \pi)a + (R_f + \pi - R)b_f - Rm - c - \tau, \]  

where \( \dot{a} \equiv (M + B + Eb_f)/P = m + b + b_f = \) total real wealth. The agent’s objective then is to maximize equation (2) subject to equations (17) and (4). By the same calculation in section 2, the optimum conditions for the representative agent are equations (6a)-(6d), (17), (4), (7), and

\[ R = R_f(b_f) + \pi. \]  

Equation (18) expresses the non-arbitrage condition of portfolio selection between domestic and foreign bonds.

As for the government sector, monetary policy is the same as equation (8). The government’s budget constraint in real value is now revised as\(^3\)

\[ \dot{a} = (R - \pi)a + (\pi - R)b_f + \dot{b}_f - Rm - \tau. \]  

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\(^2\) The flow budget constraint of the representative agent in nominal value is given by

\[ M + \dot{B} + E\dot{b}_f = Py + RB + ER\dot{b}_f - P_c - Pr. \]

Using equation (1) with \( P_f = 1 \), the definitions of \( m, b, \) and \( \pi \) in the previous section, and the definition of \( a \) in equation (17), we can obtain equation (17).

\(^3\) The flow budget constraint of the government in nominal value is given by \( M + \dot{B} = RB - Pr. \) Following the same procedure as in footnote 2, we can yield equation (19).
Combining equations (17) and (19) yields the balance-of-payments equilibrium:

\[ \dot{b}_f = y - c + R_f(b_f)b_f. \]  

Equation (20) describes that the rate of accumulation of foreign bonds is equal to the current-account surplus, which equals the trade surplus plus interest payments on foreign bond holdings. Under flexible exchange rates, the current-account surplus (deficit) is offset by the capital-account deficit (surplus) and hence the balance of payments is always in equilibrium at all times. Given the above assumptions and derivations, equations (6a)-(6d), (4), (5), (7), (8), and (18)-(20) therefore solve the perfect-foresight equilibrium of a small open economy with international capital mobility.

Manipulating equations (6a)-(6c), (4), (5), (8), and (18) gives

\[ u_c(c, 1 - g(m/c)c) = \lambda + u_c(c, 1 - g(m/c)c)[g(m/c) - g'(m/c)m/c], \]  

\[ \lambda \psi(\pi) = - u_c(c, 1 - g(m/c)c)g'(m/c), \]  

\[ \psi(\pi) = R_f(b_f) + \pi. \]

From equations (21a)-(21c), the instantaneous relationships can be derived as:

\[ \pi = \pi(b_f), \]  

\[ c = c(\lambda, b_f), \]  

\[ m = m(\lambda, b_f), \]

where \( \pi_{b_f} = R_f/\psi(\pi' - 1), c_{\lambda} = [(g')^2u_{ii} - u_{i}g^* \cdot 1/\c] + R\theta)/\Delta < 0, c_{b_f} = \lambda \psi/\pi_{b_f} \theta/\Delta, \)

\[ m_{\lambda} = \{R[u_{cc} - 2u_{cl}(g - g' \cdot m/c) + u_{cl}(g - g' \cdot m/c)^2 - u_tg^* \cdot m^2/c^2] + \theta\}/\Delta < 0, \]

\[ m_{b_f} = \lambda \psi/\pi_{b_f} \{u_{cc} - 2u_{cl}(g - g' \cdot m/c) + u_{cl}(g - g' \cdot m/c)^2 - u_tg^* \cdot m^2/c^2\}/\Delta, \]

\[ \theta = g[u_{cl} - u_{il}(g - g' \cdot m/c)] - u_tg^* \cdot m^2/c^2 < 0, \]

\[ \Delta = (g')^2(u_{cc}u_{ll} - u_{cl}^2) - u_tg^* \cdot 1/c (u_{il}g^2 + u_{cc} - 2u_tg) > 0. \]

Substituting equation (18) into (6d) and equation (22b) into (20), we have

\[ \dot{\lambda} = \lambda[\rho - R_f(b_f)] = \Gamma(\lambda, b_f), \]
\[ \dot{b}_f = y - c(\lambda, b_f) + R_f(b_f)b_f = \Omega(\lambda, b_f), \]  
\[ (23b) \]
where \( \Gamma_{\lambda} = 0, \quad \Gamma_{b_f} = -\lambda R'_f > 0, \quad \Omega_\lambda = -c_\lambda > 0, \quad \text{and} \quad \Omega_{b_f} = R_f + R'_f b_f - c_{b_f}. \) Let \( \alpha_1 \) and \( \alpha_2 \) be the two characteristic roots satisfying equations (23a) and (23b), we then have
\[ \alpha_1, \alpha_2 = -c_\lambda \lambda R'_f < 0. \]  
Equation (24) indicates that the dynamic system possesses one positive root and one negative root. Because the economy has one non-predetermined variable, \( \lambda \), and one predetermined variable, \( b_f \), there exists a unique saddle-path leading to the stationary equilibrium. Therefore, we establish the following proposition:

**Proposition 2.** In a small open economy with international capital mobility, the steady state equilibrium displays determinacy regardless of interest-rate feedback rules.

Proposition 2 runs in sharp contrast with Proposition 1. The key to this contrast is that an integrated world capital market plays an important role in facilitating the financing needs of a small open economy from abroad. Hence, the non-arbitrage condition of the agents’ portfolio choice pins down the function of the interest-rate feedback rule.

4. Concluding remarks

This paper develops a monetary model of the small open economy operating under flexible exchange rates, assumed either with or without international capital mobility. Money is introduced into the economy through the channel of shopping-time technology. Within this model, we make a new attempt to investigate the role of capital mobility in affecting the relationship between interest-rate feedback rules and macroeconomic stability. We find that, in a small open economy without capital mobility the active interest-rate rule yields a determinate equilibrium, while the passive rule offers an indeterminate equilibrium. By contrast, in a small open economy with capital mobility, the equilibrium always displays determinacy regardless of interest-rate feedback rules.
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