On material removal capability of abrasive particle in polishing process

Y.-T. Su*, T.-C. Hung† and C.-M. Tsai‡

*Department of Mechanical Engineering, National Sun Yat-Sen University, Kaohsiung, 804, Taiwan, ROC.
E-mail: yaoten@mail.nsusu.edu.tw
E-mail: shapirotsai@yahoo.com.tw

†Department of Mechanical Engineering, Chinese Military Academy, Kaohsiung, 830, Taiwan, ROC.
E-mail: Tch@cc.cma.edu.tw

Abstract: The material removal capability of an abrasive particle, with a dimension of tens of nanometre, in a polishing process was examined in this study. It was aimed to characterize the material removal capability of the particle when it moved in a certain speed relative to work or tool. A model was first proposed to describe the necessary condition that material removal could occur at the particle-work or particle-tool interfaces. Then, a set of equations was derived, according to the law of force equilibrium and the principle of minimum potential energy, to model the steady-state motion of particle. The simulations indicated that the particle speed relative to work or tool should exceed a certain value to cause a successful removal action. In addition to the surface energy of work or tool, this value depended on the adhesive works of particle at its interfaces. The study also revealed that the trend of material removal rate at work vs. tool speed or normal load was not linear. To apply the Preston equation, one should be more cautious. Finally, the trend of material removal rate at work was found complex and was inconsistent with that of wear rate at tool, under different adhesion conditions at the interface of particle. For instance, the material removal rate always increases as the normal load increases. However, the wear rate at tool may not be always proportional to the normal load.

Keywords: energy of adhesion, material removal rate, polishing process, tool wear, wear model.


Biographical notes: Yaw-Terng Su is a professor in department of mechanical engineering at National Sun Yat-Sen University. His current research interests include floating polishing methods, ultra precision machining methods, and rolling bearing vibration analysis. Su received his BSME degree from National Taiwan University in 1980. He received his master and PhD degrees from University of Wisconsin–Madison in 1984 and 1986, respectively.

Tu-Chieh Hung is an assistant professor in department of mechanical engineering at Chinese Military Academy. His current research interests include hydrodynamic polishing process and nano-technology. Hung received his PhD degree from National Sun Yat-Sen University in Taiwan in 2001.
1 Introduction

The material removal capability of an abrasive particle, with a dimension of tens of nanometre, in a polishing process is investigated. It is to understand how the particle moves relative to work or tool when driven by the moving tool. Especially, the influence of this relative motion on the material removal at work or tool surface is analysed. One goal of this study is to realize the effect of adhesions at the particle-work and particle-tool interfaces on the removal capability of particle. The other one is to characterize the removal capability of particle under various operating conditions. It is hoped that some useful rules can be induced from the study to help improving the polishing quality or guiding the selection of particle or tool material for polishing a specific work material.

The characterization of particle’s removal capability is important to make a polishing process predictable. This predictable feature is a necessity to treat the manipulation of polishing process as a technique instead of an art. It is also the very key to allow it to become an ultra-precision machining tool [1, 2]. In fact, a polishing process is notorious for its randomness in behaviours. Its random nature partly comes from the uncontrollable particle distribution between work and tool (or pad) and partly from the inconsistent removal capability of particles. There are several means were proposed to regulate the particle distribution. A floating or semi-floating polishing process [3–5] was shown effective to control the particle distribution. A proper groove design on the pad of CMP process is also believed useful. However, the time-varying nature of volume removal rate was always observed in a polishing process [6–10]. The tool (or pad) wear was postulated to be the main cause of the phenomenon. Such a wear was resulted from the action of abrasive particle on tool surface. The particle’s removal capability was then influenced by the worn tool. This interacting loop will complicate the behaviours of a polishing process. The prediction of polishing behaviours without simultaneously considering the wear effect may not be accurate. Therefore, it is equally important to model the removal capability of particle on tool as that on work.

In the past decades, several models [11–31] had been proposed to describe the volume removal rate of a polishing process. Most of them were heuristic and were based on some rules in deriving their mathematical forms. One of the most adopted rules was the wear rate expression for adhesive wear, proposed by Archard [11]. The other one is the rule for abrasive wear [12, 30]. The Preston’s equation [13] was the mostly referred rule in describing the overall removal rate of the CMP process. Runnels et al. [14–16] suggested that the removal rate of the CMP process should be decided by not only the normal stress but also the shear stress. Cook’s model [17] dealt with the mechanics of abrasive particles and with the chemical reactions in glass polishing. The effect of pad’s surface irregularities on removal rate were examined by Yu et al. [18] and Su et al. [19]. In addition to the effort of modelling
CMP process, there were many studies aimed at the modification of the Preston equation. By using a microscopic analysis of polishing mechanism, Zhao and Shi [20] suggested that there should exist a threshold pressure in CMP process. Zhang et al. [21] derived a model including the effects of polishing pressure and platen speed on particle penetration depth in CMP process. Recently, a model for mechanical wear and abrasive particle adhesion in CMP process was studied by Ahmadi and Xia [22]. Besides the CMP process, the removal rate characteristics of the HDP process (hydrodynamic polishing process, a floating process) were extensively studied by Su et al. [2, 5, 8, 10]. Many of the conclusions made from the HDP process could be also applied to the CMP process. To have a more full outlook on polishing process, one can refer to the wonderful article written by Komanduri, et al. [12]. A more extensive survey about the CMP process is available in the study by Nanz and Cammilleti [23]. The existing CMP models were also widely reviewed by Tseng et al. [24].

The above models mostly assumed that the wear mechanism of a polishing process should be abrasion or adhesion. In the abrasive wear, it was generally adopted that the wear rate was proportional to the penetration depth of particle and the relative speed between work and tool. The wear rate of adhesive wear was assumed proportional to the real contact area and relative speed. If these rules were used in modelling the removal capability of a tiny abrasive particle (with a dimension of tens of nanometre or less), two questions might be raised. First, will the penetration depth be the removal depth in the abrasive wear even if the penetration depth is as small as several nanometres or less? Secondly, should the material be removed in the adhesive wear no matter how slow the relative speed could be? It is believed that the real factor in deciding the wear rate is the relative speed between particle and work but not the one between tool and work, although the first speed is proportional to the latter one. The relative speed between particle and work is a result of mutual actions of the forces that are occurred at the interfaces of particle with work and tool. Many factors should influence these forces, such as the deformation of tool or work. The adhesion between particle and work or tool is another factor. A simple assumption made on the relative speed, without incorporating these forces in describing the particle motion, might be inaccurate. Further, to have material removal, a sufficiently large force should exist at the interface of particle to pull the surface atoms away. This pull force is also related to the interfacial reaction of particle. It should not be necessarily that the pull force is always sufficiently large. Accordingly, the use of abrasive or adhesive rule in modelling the removal capability of particle must be cautious.

In this study, a model is proposed to describe the relative motion of abrasive particle in a polishing process. The attention will be focused on estimating the pull force in tearing the surface atoms. Then, the removal capability of particle on work or tool surface under various conditions is analysed.

2 A force model for abrasive particle

The kinematics of an abrasive particle, under the action of tool against work surface, was first modelled in this section. The analytic approach in studying the rolling friction by Kendall and others [31, 33] was adopted in the following derivation. The previous
studies [31] proposed that a certain amount of energy must be supplied to an object to make it rolling on a surface. This energy should come from the action of an external force and will result in a resistant force (or friction force) at the interface between object and work. The magnitude of the force or energy depended on not only the adhesive surface energy [32] between object and work but also the rolling speed of object. The nature of this mutual interaction between resistant force and rolling motion will constitute the kinematics of abrasive particles in a polishing process.

Consider a small spherical particle (say, with a radius \( r \) less than 0.05 \( \mu m \)) rolling on a surface, under the press of a normal load (Figure 1). Let the contact spot, between particle and surface, be a circle with diameter \( a \). When the particle rolls over the surface, the tail edge of particle will break away from the surface (called breaking process) and the front edge will mate with the surface (making process). Because of the adhesion between particle and surface, a sufficient energy should be supplied to the particle to have the breaking process. On the other hand, some energy will be released in the making process. The energy needed in the breaking process is always larger than the released energy in the making process. Thus, to have a rolling motion, an external energy must be applied to the particle. If the particle rolled a distance \( s \), the energy supplied in such a motion can be expressed as

\[
\text{supplied energy in the rolling motion} = F \cdot s = a \cdot s \cdot e_r
\]

(1a)

where \( F \) is the external force applied at the particle or equivalently the friction force from the surface. The product \( a \cdot s \) is the area rolled over by the particle. \( e_r \) is the energy required if a unit area of surface is rolled over. This energy depends on the energy released or needed in the making or breaking process of rolling motion [31] and is function of the adhesive surface energy between particle and surface. The Kendall’s study [31] on friction indicated that the needed energy in the breaking process could be sensitive to the relative speed between the tail edge of particle and the surface. It suggested that \( e_r \) was positively related to the rolling speed in

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**Figure 1** A schematic diagram of a spherical particle under the press of a normal load.
a moderate range of speed. Hence, Equation 1 can be rewritten as

\[ F = a \cdot e \cdot r \propto \omega \]  

(1b)

when the particle moves at an angular speed \( \omega \).

If the particle has a steady-state rolling motion, it must be in the force-equilibrium condition. In this condition, the external applied force and friction force will form a couple (with magnitude \( F \cdot r \)). Hence, the adhesion distribution at the interface between particle and surface (Figure 2a) should induce an equivalent moment to counterbalance the couple. This means that the surface will encounter a similar but with an opposite direction of adhesion distribution (Figure 2b). The strength of this adhesion is proportional to the magnitude of couple. Because of this adhesion, some of the surface atoms are drawn away from their equilibrium positions. The stronger the adhesion is the larger the separation of atoms will be. To simplify the following analysis, this adhesion distribution is represented by another couple that

\[ F' \cdot b = F \cdot r. \]  

(2)

![Diagram showing force distribution](image)

*Figure 2* A schematic diagram of the adhesion distribution at the interface between particle and surface.
The dimension of $b$ depends on the shape of adhesion distribution. In Figure 2b, the shape of adhesion distribution is obtained from three assumptions. Firstly, it is assumed that the energy or force required for the particle to separate from or approach to surface atoms is linearly proportional to its peeling or approaching speed. Secondly, it is assumed that the rolling centre is located at the centre of contact spot. Finally, the adhesion outside the contact spot is negligible. According to these assumptions, half of contact spot tends to pull the surface atoms and the other half presses the atoms. As a result, the resultant $F'$ of adhesion distribution is located at the centroid of each half of contact spot and the distance $b$ is equal to $\frac{4a}{3\pi}$. Let the work done by the pull force $F'$ be completely used to separate the atoms. If this work is less than the required energy to separate the surface layer and its sub-layer atoms, no material removal occurs and the following inequality is satisfied,

$$F' < a \cdot 2 \cdot \gamma$$  \hspace{1cm} (3)$$

where $\gamma$ is the surface energy of surface atoms.

However, from Equation 1b, the force $F'$ may be enhanced to violate Equation 3 when the rolling speed of particle is increased. In that case, the force $F'$ is sufficiently large to pull the surface atoms away and result in material removal. From the study by Kendall [32], one may expect that the surface energy $\gamma$ is also proportional to the separating speed between atoms. Thus, one may obtain the following equation under the steady-state condition

$$F' = a \cdot 2 \cdot \gamma (v')$$  \hspace{1cm} (4)$$

where $v'$ represents the average speed of surface atoms leaving the subsurface ones. This implies that the surface layer of atoms has a peeling motion with speed $v'$ while the particle rolls over the surface layer. A schematic diagram of this motion is shown in Figure 3. Assume the rolling centre of particle is located at the centre of the contact spot. The average upward speed of particle at its tail edge is $\frac{5a}{\pi} \cdot \omega$. The average relative speed between particle's tail edge and surface atoms in the breaking process becomes $\frac{5a}{\pi} \cdot \omega - v'$. Then, the energy required to have a unit area of rolling is $\epsilon r (\frac{5a}{\pi} \cdot \omega - v')$. When material removal occurs, the energy or power supplied to the

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3}
\caption{A schematic diagram of the peeling motion of surface atoms.}
\end{figure}
particle will be partly used in its rolling motion and the rest is employed in material removal. The required power to have a rolling speed \( \omega \) can be then written as

\[
F \cdot r \cdot \omega = a \cdot (r \cdot \omega - v') \cdot e_t \left( \frac{2d}{3\pi} \cdot \omega - v' \right) + a \cdot v' \cdot 2 \cdot \gamma(v').
\]  

(5)

In addition, from Equations 1, 2, 4 and 5, \( e_t \) and \( \gamma \) must satisfy the following equation

\[
\left(1 - \frac{v'}{r_0 \omega}\right) \cdot e_t \left( \frac{2d}{3\pi} \cdot \omega - v' \right) = \left( \frac{4d}{3\pi r} - \frac{v'}{r_0 \omega} \right) \cdot 2 \cdot \gamma(v').
\]

(6)

Equation 5 shows that the power consumed in material removal is simultaneously decided by \( v' \) and \( \gamma \). However, Equation 6 implies that \( v' \) and \( \gamma(0) \) tend to have opposite trends. A large value of peeling speed \( v' \) always requires a small value of \( \gamma \). Similarly, a large value of \( \gamma \) will result in a small value of \( v' \).

In practice, the particle may have both the rolling and sliding motions on the surface. Assume the effects of rolling and sliding motions on friction are independent. Then, the energy needed for the particle to travel a distance \( s \), without the occurrence of material removal, by a specific rolling plus sliding motion is

\[
F \cdot s = a \cdot s_t \cdot e_t + a \cdot s_s \cdot e_s
\]

(7)

with \( s = s_t + s_s \)

where \( s_t \) or \( s_s \) represents the travelling distance of particle due to rolling or sliding motions, respectively. \( e_s \) is the energy required if a unit area of surface is swept over by sliding motion. The first or second term in the right-hand side of Equation 7 represents the energy needed in rolling or sliding the particle a distance \( s_t \) or \( s_s \), respectively. Under the steady-state condition, Equation 7 can be rewritten as

\[
F \cdot v_p = a \cdot (r \cdot \omega) \cdot e_t + a \cdot v_s \cdot e_s
\]

(8)

where \( v_p \) is the speed of particle at its mass centre. \( (r \cdot \omega) \) or \( v_s \) is the rolling or sliding speed of particle relative to the surface, respectively.

Equation 8 becomes improper if the rolling or sliding motion causes the separation of the surface and its sub-layer atoms. It can be happened because both \( e_t \) and \( e_s \) are function of rolling or sliding speed. As described above, part of the adhesion distribution at the interface between particle and surface tends to pull the surface atoms away. The strength of adhesion is enhanced when the rolling or sliding speed increases. Hence, a sufficiently high speed may result in material removal. By some manipulations, one can find that the necessary criterion to have material removal is as follows

\[
\left( \frac{r_0 \omega}{v_p} \cdot e_t \left( \frac{2a \alpha}{3\pi} \right) + \frac{v_s}{v_p} \cdot e_s(v_s) \right) \cdot \left( \frac{3\pi r}{4a} \right) > 2 \cdot \gamma(0).
\]

(9)

Once the inequality Equation 9 is satisfied, the surface atoms may be pulled away due to particle's motion. Under this condition, the power consumed in the process
can be written as

\[ F \cdot v_p = a \cdot (r \cdot \omega - v') \cdot e_t \left( \frac{2a}{3\pi} \cdot \omega - v' \right) + a \cdot v_s \cdot e_s(v_s) + a \cdot v' \cdot 2 \cdot \gamma(v'). \]  \hspace{1cm} (10)

The first two terms in the right-hand side of Equation 10 denote the power lost in the specific motion and the third one is the power used in material removal. Because the surface atoms are peeled in a direction normal to the surface, the peeling speed \( v' \) is assumed to have little effect in the making and breaking process of sliding. The second term in the right-hand side of Equation 10 is then not a function of \( v' \).

Under the steady-state condition, the peeling speed \( v' \) should balance the pull force \( F' \) and the sub-layer atoms' resistance \((a \cdot 2 \cdot \gamma(v'))\) (Equation 4). Hence, from Equations 2 and 10, the \( v' \) satisfies the following equation

\[ \left[ \frac{\nu_0}{v_p} \right] \cdot e_t \left( \frac{2a}{3\pi} \cdot \omega - v' \right) + \frac{v_s}{v_p} \cdot e_s(v_s) = \left( \frac{4a}{3\pi} \frac{v'}{v_p} \right) \cdot 2 \cdot \gamma(v'). \]  \hspace{1cm} (11)

Again, the power used in material removal is decided by peeling speed \( v' \) and surface energy \( \gamma \). A detailed investigation of Equation 11 will show that \( v' \) and \( \gamma \) have opposite trends as the case of pure rolling.

There is another extreme case that Equations 10 and 11 cannot be used to describe the particle motion and material removal process. It happens when the adhesion between particle and surface is stronger than that between surface and sub-layer surface atoms, at any motion condition. In another word, Equation 9 is satisfied even \( \omega \) and \( v_s \) are zero. Under this condition, any particle motion will immediately cause the removal of surface material. The instantaneous peeling speed of surface atoms is linearly proportional to the particle speed. Most of the energy or power consumed in the process is used in material removal. However, the relationship between particle motion and the energy or power used in this motion becomes indeterminate. The elastic emission machining, proposed by Mori [1], belongs to this case.

In a polishing process, an abrasive particle may be pressed by tool to have contact with tool and work surfaces at the same time. This particle will then have certain motions relative to tool and work when driven by the tool's or work's motion (Figure 4). Because of these relative motions, the particle will sustain two friction forces. One friction force is from the interfacial interaction between particle and work. The other one comes from the mutual action between particle and tool. If a steady-state motion of particle can be maintained, these friction forces must be in equilibrium. There are three possibilities in the steady-state motion. Firstly, the particle is stuck on the tool and has a pure sliding motion relative to work surface. The grinding machining belongs to this case. The second one is the converse of the first one that the particle sticks to the work surface. No wear (or material removal) on work surface is possible in this case. The last one is the case that the particle has a specific motion relative to both tool and work. Material removal on tool or work surface may occur if certain conditions are satisfied (will be discussed below). In the following study, only the third case is examined. To simplify the following notations, let the motions of particle and tool be all measured relative to work. Assume the tool has a speed \( v_p \). When excluding the extreme cases as discussed above,
from Equations 9, 10 and 11, the following equations should be simultaneously satisfied.

\[
\begin{align*}
& w d \cdot \left( \frac{r_0 - w v'}{r_0 + w v_s} \right) \cdot w e_r \left( \frac{2 \pi a}{3 \pi} \cdot \omega - w v' \right) + w d \cdot \left( \frac{w v_s}{r_0 + w v_s} \right) \cdot w e_s \left( w v_s \right) \\
& + \omega d \cdot \left( \frac{w v'}{r_0 + w v_s} \right) \cdot 2 \omega \gamma \left( w v' \right) \\
& = i a \cdot \left( \frac{r_0 - v}{r_0 + v} \right) \cdot i e_r \left( \frac{2 \pi a}{3 \pi} \cdot \omega - v \right) + i a \cdot \left( \frac{v}{r_0 + v} \right) \cdot i e_s \left( v \right) \\
& + i a \cdot \left( \frac{v}{r_0 + v} \right) \cdot 2 i \gamma \left( v \right)
\end{align*}
\]

= friction force \( F \) from work or tool,

\[
\begin{align*}
& \left[ \frac{(r_0 - w v')}{r_0 + w v_s} \cdot w e_r \left( \frac{2 \pi a}{3 \pi} \cdot \omega - w v' \right) + \frac{w v_s}{r_0 + w v_s} \cdot w e_s \left( v_s \right) \right] \\
& \left[ \frac{4 \pi a}{3 \pi r} - \frac{w v'}{r_0 + w v_s} \right] \cdot 2 \cdot w \gamma \left( w v' \right) \quad (12) \\
& \left[ \frac{(r_0 - v)}{r_0 + v} \cdot i e_r \left( \frac{2 \pi a}{3 \pi} \cdot \omega - v \right) + \frac{v}{r_0 + v} \cdot i e_s \left( v \right) \right] \\
& \left[ \frac{4 \pi a}{3 \pi r} - \frac{v}{r_0 + v} \right] \cdot 2 \cdot i \gamma \left( v \right) \quad (13)
\end{align*}
\]

with \( v_0 = w v_s + 2r_0 \omega + v_k \). When ignoring the left-hand side sub-index, the physical meanings of the above notation are the same as discussed before. The \( t \) and \( w \) of the left-hand side sub-indices denote the tool and work, respectively. Since the particle is assumed to have
a dimension as small as several tens of nanometre, the dimension of contact spot \( w_a \) or \( t_a \) can be modelled by the JKR equation \([33, 34]\)

\[
q_a = \frac{p \cdot r}{K} - \frac{r \cdot \left( 3\pi \eta_{cr} + \sqrt{6\pi \eta_{cr} r + (3\pi \eta_{cr} r)^2} \right)}{K} \right)^{1/3}
\]

where \( p \) is external normal load, \( K \) is the elastic parameter (defined from Young's modulus \( E \) and Poisson's ratio \( \nu \) of the elastic body that \( K = (4/3) \cdot E/(1 - \nu^2) \)) and \( q_{er} \) is the thermodynamic work of adhesion.

Equations 13 and 14 are proper if the following two inequality equations

\[
\left[ \frac{r \omega}{r \omega + w v_s} \cdot w e_r \left( \frac{2w}{3\pi} \cdot \omega \right) + \frac{w v_s}{r \omega + w v_s} \cdot w e_s \left( w v_s \right) \right] - \left[ \frac{3\pi r}{4a_d} \right] > 2 \cdot w \gamma(0) \tag{16}
\]

and

\[
\left[ \frac{r \omega}{r \omega + t v_s} \cdot t e_r \left( \frac{2t}{3\pi} \cdot \omega \right) + \frac{t v_s}{r \omega + t v_s} \cdot t e_s \left( t v_s \right) \right] - \left[ \frac{3\pi r}{4a_d} \right] > 2 \cdot t \gamma(0) \tag{17}
\]

are satisfied. These two inequality equations are the necessary criteria that material removal occurs at both the work and tool. If they are not fulfilled, no material removal occurs and \( w \cdot v' \) and \( t \cdot v' \) are equal to zero.

To characterize the motion and removal rate of particle, four unknowns need to be solved, which are \( \omega, w v_s, w w \cdot v' \) and \( t v' \). However, there are only three equations (Equations 12, 13, and 14) available in governing the particle motion and its interaction with work or tool atoms. Thus, an infinite number of solutions can meet the equations. It is similar to the statically indeterminate problem of a rigid structure because only the force equilibrium is considered. To find a unique solution, the principle of minimum potential energy \([35]\) may be used. If only the potential energy of peeled atoms is considered, one can find that to minimize the potential energy is equivalent to minimize the friction force at the interface of particle. Accordingly, the following condition should be fulfilled to find the solution

Friction force \( F \) is minimized. \( \tag{18} \)

Equations 12, 13, 14 and 18 constitute the governing equations of particle motion and volume removal rate of work or tool when the particle is pressed and driven by tool.

3 Removal rate of abrasive particle

Based on the above governing equations, the removal capability of a particle under the direct press of tool is examined in this section. The effect of different operating conditions, such as tool speed change or normal load variation, on removal capability will be studied by the computer simulations. The influence of adhesion at the particle-work and particle-tool interfaces is especially noted. From the previous derivation, only one layer of atoms is removed at each removal action. The volume removal rate of particle is equal to the product of three terms,
which are the contact dimension $a$, the peeling speed $v'$ and the diameter of removed atoms. Since the diameter of atom is fixed for a given material, the trend of volume removal rate is the same as that of $a \cdot v'$. In the following study, the removal capability of particle is represented by $a \cdot v'$ and is denoted as the (material) removal rate or wear rate of particle. To better understand the removal rate, the relationships between peeling speed and various affecting factors will be also analysed. Both the material removal rate at work surface and the wear rate at tool surface are investigated.

Because of the non-linear nature of governing equations, the dimensionless analysis of removal rate is not available in this study. Those cases that result in pure particle motion without material removal will not be examined. Further, the extreme cases, with particle firmly adhered to work or tool surface, are also excluded. The energy required in sliding motion is always assumed larger than that in rolling motion. Two cases are examined. The first case (denoted as Case I) has the material properties that the adhesion between particle and work is stronger than that between particle and tool. That is the condition of $\omega a \cdot \omega e_r > a \cdot v e_r$ and $\omega a \cdot \omega e_s > a \cdot v e_s$ is satisfied. The other one (denoted as Case II) has the opposite relationship. It is also assumed that $e_r$ or $e_s$ is proportional to $\omega$ or $v_s$, respectively. According to the experimental data of Kendall’s study, they should have a non-linear and saturated trend at a moderate speed range as indicated in Figure 5.

In the following simulations, the particle has a spherical shape with diameter 50nm. The work surface is a flat one. The moduli of tool and work are $1.0 \times 10^7$ and $1.29 \times 10^{11}$ N/m$^2$, respectively. Without relative motion, the adhesive works of particle at the interfaces with work and tool are assumed to be linearly proportional to $\omega e_r(0)$ and $v e_r(0)$. The work is assumed stationary and the tool moves at a speed $v_s$. The simulated parameters for the two cases are listed in Table I.

The effect of tool speed on particle motion for the two examined cases can be generally described by Figures 6 and 7. Two features are noted. Firstly, the particle tends to have a pure rolling motion relative to work and a hybrid motion (rolling plus sliding) relative to tool for Case I. The trends of relative motion are reversed for Case II. If the simulated parameters are so changed that the condition of Case I is transferred to Case II, the feature of particle motion at the interfaces will make

<table>
<thead>
<tr>
<th>Table I</th>
<th>The simulated parameters for Cases I and II.</th>
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<tbody>
<tr>
<td></td>
<td>$\alpha_t$ (J/m$^2$)</td>
</tr>
<tr>
<td>Case I</td>
<td>Tool 0.015</td>
</tr>
<tr>
<td></td>
<td>Work 1</td>
</tr>
<tr>
<td>Case II</td>
<td>Tool 0.2</td>
</tr>
<tr>
<td></td>
<td>Work 1</td>
</tr>
</tbody>
</table>

Rolling adhesion energy $\alpha_t + \beta_t \cdot Q_t \cdot \tan^{-1}\left(\frac{\nu_s}{Q_t}\right)$

Sliding adhesion energy $\alpha_s + \beta_s \cdot Q_s \cdot \tan^{-1}\left(\frac{\nu_s}{Q_s}\right)$

Surface energy $\alpha_t + \beta_t \cdot v'$
Figure 5  A schematic representation for the relationship between \( e_r \) or \( e_s \) and \( r \).

(a) Particle speed relative to work surface (m/s)

(b) Particle speed relative to tool surface (m/s)

Figure 6  Effects of tool speed on particle motion for Case I, (a) Particle speed relative to work, (b) Particle speed relative to tool.
the corresponding transition. Secondly, the rolling or sliding speed of particle at the interfaces tends to be linearly proportional to the tool speed. This linear trend is insensitive to the non-linear nature of $e_t$ or $e_s$.

The peeling speed $\dot{w} v'$ vs. tool speed for Case I is shown in Figure 8. It reveals a non-linear trend. In addition, no material removal is possible if the tool speed is less than a critical one. This non-linear increasing trend of peeling speed at work surface is related to the non-linear trend of $\dot{w} e_t$. From Figures 6 and 7, the increase of particle speed is linearly proportional to the tool speed increment. However, the increase of peeling force, for a given increment of tool speed, will be lessened at high speed because of the saturated feature of $\dot{w} e_t$. Thus, a saturated tendency of removal rate vs. tool speed is obtained. The incapability of material removal at low tool speed is due to the violation of Equation 9. Thus, a dominant factor in determining the critical speed is the surface energy of work surface. The relationship between critical speed and
Figure 8  The peeling speed $u_Y$ vs. tool speed for Case I.

Figure 9  The relationship between critical speed and surface energy of work for Case I.

Figure 10  Effect of work’s surface energy on peeling speed at work surface.
surface energy of work for Case I is presented in Figure 9. The $\gamma_w$ in this figure is the surface energy of work for Case I, whose value is determined by the parameters listed in Table I. Then, the ratio $\frac{\gamma}{c_{w}}$ denotes the relative enhancement of work's surface energy. It indicates that the critical speed will be increased with an increment of surface energy. A similar result can be also obtained for Case II. The effect of work's surface energy on peeling speed can be further examined in Figure 10. A decreasing trend between peeling speed and surface energy is obtained for Case I. This demonstrated an well-accepted result that the higher the surface energy of work is the lower the removal rate will be. A similar trend can be also obtained for Case II.

The influence of adhesive strength at the particle-work and particle-tool interfaces on peeling speed and removal rate can be generally represented by Figures 11, 12, 13, and 14. It reveals that an increase of $c_{w}$ always increases the

![Graph 1](image1.png)

**Figure 11** Effect of adhesive strength $c_{w}$ at the particle-work and particle-tool interfaces on peeling speed, (a) Case I, (b) Case II.
Figure 12 Effect of adhesive strength $\gamma_{ct}$ at the particle-work and particle-tool interfaces on peeling speed, (a) Case I, (b) Case II.

peeling speed of tool (Figure 11). On the other hand, the $w_{ct}$ has a negative effect on work’s peeling speed in both cases. If the removal rate is examined, the wear rate at tool increases in both cases as $w_{ct}$ is enhanced. However, the material removal rate has an increasing trend in Case I and a decreasing trend in Case II. The increasing trend of wear rate at tool is due to the increase of peeling speed. The decreasing trend of material removal rate in Case II is also from the decrease of peeling speed. However, the trends of peeling speed and material removal rate (at work) are found inconsistent in Case I. It is because in Case I the increase of contact dimension $\omega d$ exceeds the decrease of $w_{ct}$ when $w_{ct}$ increases. This phenomenon can be explained as follows. In case 1, the adhesion between particle and work is higher than that between particle and tool. To drive the particle moving over the work surface, the particle-tool interface needs a sliding motion to create a sufficiently large force, because the value of $\gamma_{ct}$ is larger than that of $\gamma_{ct}$. Once the adhesion at the particle-work
interface is further strengthened, to keep force balance and maintain the lowest the interfacial force, the particle speed relative to work should reduce. However, the interfacial force needs further increased to drive the particle to resist the strengthen adhesion at particle-work interface. This will result in a relatively slight decrease of peeling speed at work surface. Because the contact spot at the particle-work interface enlarges obviously as \( w_{c} \) enhances, the product \( w_{a} \cdot w_{v} \) increases. The above explanation can be also used to explain the other trends of \( w_{v} \) and \( r_{v} \). It is noted that the effect of \( w_{c} \) on \( w_{v} \) or \( r_{v} \) for Case I and II is similarly to the effect of \( w_{c} \) on \( r_{v} \) or \( w_{v} \) for Case II and I, respectively.

The effect of normal load on peeling speed and removal rate can be generally denoted by Figures 15 and 16. It indicates that the peeling speed at work increases as the load increases for both cases. This increasing trend becomes less significant when the load is high. Similarly, the removal rate at work has an increasing but non-linear
trend. The increasing trend of removal rate is more obvious than that of peeling speed at high load. This suggests that the increase of removal rate, at high load, is mainly due to the increase of contact spot, but not the enlargement of peeling speed. In contrast to work's peeling speed, the normal load is shown to have negative effects on the peeling speed at tool. The wear rate at tool has a decreasing trend in Case I but an increasing one in Case II. The effect of modulus on removal rate is shown in Figures 17 and 18. It indicates that the material removal rate at work is little affected by the change of tool's or work's modulus in Case I. It becomes positively related to work's modulus and little affected by tool's modulus in Case II. On the other hand, an increase of work's modulus will reduce the wear rate at tool in both cases. The wear rate enhances if the tool's modulus increases in Case I. The influence of tool's modulus on wear rate is negligible in Case II.
4 Discussions

There are several points can be summarized from the above study. Firstly, it is possible that the peeling motion (or removal) of surface atoms can happen when the static adhesive work between particle and work surfaces (without relative motion) is less than work's surface energy. It is because the adhesive work at interface will drastically increase with speed increment in the breaking process. Hence, as long as the relative speed between particle and work is high enough that the dynamic adhesive work exceeds the surface energy, material removal may occur. This point implies that a relative motion between particle and work does not necessarily bring about material removal. The tool speed, which is the driving source of particle motion, should exceed a critical value to cause machining action at work. A sufficiently large tool speed is to have enough peeling force $F_p$ to tear the work atoms. Hence, any factor that can enhance the peeling force or reduce the resistance...
force from work, for a fixed area of work surface, can decrease the critical speed. The simulations revealed that the critical speed was proportional and sensitive to the surface energy of work. In general, an increment in the adhesion between particle and work can lower the critical speed. Secondly, both the tool speed and normal load have positive effects on the material removal rate of work. However, the removal rate is not linearly proportional to them. Especially, under high load or driven by a tool speed far beyond critical speed, the removal rate becomes less sensitive to speed or load. The nonlinear trend of removal rate is mainly due to the non-linear nature of \( w_e \) or \( w_e^2 \) and \( w_d \). This result implies that the use of the Preston equation should be confined to a small range of load and speed.

Thirdly, the removal capability of particle on work or tool was found to be affected by the variation of work's or tool's modulus. The relative dimension of contact spots and the relative adhesive strength at the interfaces are the important factors in deciding the influence of modulus. This can be explained by examining
the trends in Figure 17(a). The adhesion at the particle-work interface is stronger than that at the particle-tool interface in this example. The particle motion is dominated by the adhesion of particle-work interface. An increase of tool’s modulus will reduce the dimension of contact spot, and hence decrease the adhesive capability at the particle-tool interface. To meet the force balance between the particle-work and particle-tool interfaces, the particle speed relative to tool needs to be enlarged while the particle speed relative to work reduces. The reduction of particle speed relative to work will decrease the material removal rate at work. Because of the increase of particle speed, the increase of wear rate at tool becomes possible. Since the adhesion at the particle-work interface is higher, the speed reduction at work is less than the speed increment at tool. Hence, the wear rate variation can be more obvious than the variation of material removal rate. This explanation can be also used to illustrate the other trends in Figures 17(b) and 18. A useful rule can be induced from these trends that a soft tool (with low modulus) will result in a low wear rate at tool.
Figure 18  Effect of Young's modulus on removal rate for Case II. (a) Tool's modulus; (b) Work's modulus.

Finally, the strength of adhesion at the interfaces of particle plays an important role in particle's motion and work's or tool's wear. The relationship between adhesion and work's or tool's wear is not simple. A general trend is that an increase in the adhesive strength \( \tau_r \) between particle and tool will increase the material removal rate at work. It is because such an increment always increases the particle speed relative to work and reduces the speed relative to tool. The increase of particle speed relative to work will enlarge the friction force at its interface and hence increase the removal rate. However, the increase of \( \tau_r \) does not necessarily enlarge the wear rate at tool. When the adhesion condition belongs to Case I (strong adhesion at the interface between particle and work surface), the wear rate may decrease with the increment of \( \tau_r \). The wear rate becomes positively related to \( \tau_r \) in Case II (strong adhesion between particle and tool). The main cause of this
phenomenon is due to the non-monotonic trend of peeling speed $v'$ of tool surface. The $v'$ was shown to have a decreasing trend vs. $\epsilon_t$ in Case I and an increasing trend in Case II. Hence, the variation of wear rate becomes complex. At first sight, the increase in the adhesive strength $we_t$ between particle and work should have an opposite effect on wear rate of work or tool as that of $\epsilon_t$. The simulations did reveal that the increase in the adhesive strength $we_t$ could always increase the tool's wear rate. The non-monotonic trend also occurred to the peeling speed $wv'$ of work surface. Nevertheless, unlike the previous one, the removal rate at work was found to always increase with the increment of $we_t$. This non-consistent trend is due to the large modulus of work. It will make the dimension of contact spot sensitive to the enhancement of interfacial adhesion. Thus, the increase of $we_t$ may exceed the decrease of $v'$ in the condition of Case I. The removal rate becomes increasing as $we_t$ enhances. The above point suggests that a better polishing behaviour (high removal rate at work and low wear rate at tool) may be obtained if the adhesions at the interfaces of particle are properly adjusted.

Another phenomenon worthy of attention is the effect of normal load on the wear rate of tool. The simulations disclosed that the material removal rate at work always had an increasing trend vs. the load. However, this increasing trend did not necessarily apply to the wear rate at tool. The wear rate was found possibly to decrease as load increased if the modulus of tool is low and the adhesive strength between particle and tool has the order as that between particle and work. It because the modulus of tool usually has a magnitude several order lower than that of work. An increase of normal load will make the dimension increment of contact spot between particle and tool much greater than that between particle and work. This increment of contact dimension will enlarge the mutual adhesive strength. Thus, the enlargement of adhesive force at the particle-tool interface may become greater than that at the particle-work interface if the adhesions at different interfaces do not have an enormous difference. To keep the force balance, the particle-tool interface needs to lower the peeling speed while the particle-work interface raise its peeling speed. This results in the wear rate reduction at tool due to load increase.

There were several postulations made in the above study. In the proposed removal model, it was assumed that the removal of surface atoms could only be pulled off the surface (atoms leave in the normal direction of surface). It was not suggested that the atoms could be 'skinned off' the surface (atoms leave in parallel with surface). In addition, the law of force equilibrium and the principle of minimum potential energy were adopted in deriving the equations of particle motion. Especially, the later principle was used to deal with the non-unique problem of solution. Finally, the particle was assumed to be a sphere. The influence of particle shape on its motion was not explicitly included in the governing equations. In spite of these assumptions, the proposed model revealed some properties of polishing process, which were consistent with experimental data (for instance, the non-linear trend of removal rate vs. tool speed). The model also indicated the material removal rate at work and wear rate at tool was greatly influenced by the relative strength of adhesions of particle at the interfaces with work and tool. This point might fit to the experience in polishing different work materials by various kinds of particles. A careful verification by experiments is still needed. Above all, in addition to removal rate, the influence of relative strength to other polishing properties (such as surface roughness or surface damage) is worthy of further analysis.
5 Conclusions

The material removal capability of an abrasive particle was analysed in this study. It was done by examining the reactions at the interfaces of particle when the particle moved at a speed relative to work or tool surface. The phenomenon of material removal at work or tool surface was assumed to occur if the pull force at the interface exceeded the adhesive strength between the surface layer and its sub-layer atoms. To characterize the particle motion, a set of equations was derived according to the law of force equilibrium and the principle of minimum potential energy. The effects of various operating conditions and material properties on removal capability of particle were investigated. Especially, the influence of adhesive strength at the interfaces with tool and work, was noted.

The simulations indicated that a relative motion between particle and work or tool could result in material removal at their surfaces. It was because the adhesion at the interface could be enlarged, due to the increase of relative speed, to break the bonding between surface and its subsurface atoms. To do so, the tool speed should be higher than a critical one to have an effective material removal. This critical speed is function of the surface energy of work and the adhesive strength at the interfaces of particle. The study also revealed that the relationship between material removal rate and tool speed or normal load was non-linear. This non-linear trend is due to the non-linear nature of adhesion at the interfaces. Finally, the material removal rate at work surface or wear rate at tool surface was shown to be related to the adhesive strength at the interfaces of particle. In most cases, an enhancement at the particle-tool or particle-work interface will increase the material removal rate at work. However, such an enhancement does not necessarily increase the wear rate at tool. To have a high removal rate at work and low wear rate at tool, the adhesion at the particle-tool interface should be higher than that at the particle-work interface.

References

On material removal capability of abrasive particle in polishing process


List of symbols

\( s \)  travelling distance of a particle
\( a \)  contact diameter of a particle
\( F \)  external force applied at the particle
\( F' \)  resultant of adhesion distribution applied at the particle
\( b \)  centroid distance of adhesion distribution
\( e_r \)  energy required if a unit area of surface is rolled over
\( e_s \)  energy required if a unit area of surface is slid over
\( γ \)  surface energy of surface atoms
\( r \)  radius of a particle
\( ω \)  angular speed of the particle
\( v' \)  average speed of surface atoms leaving the subsurface ones
\( v_p \)  speed of particle at its mass centre
\( s_r \)  travelling distance of a particle due to rolling motion
\( s_s \)  travelling distance of a particle due to sliding motion
\( ω \)  rolling speed of the particle relative the surface
\( v_s \)  sliding speed of the particle relative the surface
\( v_o \)  relative speed between tool and work
\( E \)  Young’s modulus
\( p \)  external load
\( K \)  elastic parameter
\( v \)  Poisson’s ratio

\textit{List-hand side} subscripts

\( t \)  tool
\( w \)  work